6.41 \( P(1.8 < X < 2.4) = \int_{1.8}^{2.4} xe^{-x} \, dx = (xe^{-x} - e^{-x}) \bigg|_{1.8}^{2.4} = 2.8e^{-1.8} - 3.4e^{-2.4} = 0.1545. \)

6.44 (a) \( \mu = \alpha \beta = 6 \) and \( \sigma^2 = \alpha \beta^2 = 12. \) Substituting \( \alpha = 6/\beta \) into the variance formula we find \( 6\beta = 12 \) or \( \beta = 2 \) and then \( \alpha = 3. \)

(b) \( P(X > 12) = \frac{1}{16} \int_{12}^{\infty} x^2 e^{-x/2} \, dx. \) Integrating by parts twice gives

\[
P(X > 12) = \frac{1}{16} \left[ -2x^2 e^{-x/2} - 8xe^{-x/2} - 16e^{-x/2} \right]_{12}^{\infty} = 25e^{-6} = 0.0620.\]

6.45 \( P(X < 3) = \frac{1}{4} \int_{0}^{3} e^{-x/4} \, dx = -e^{-x/4} \bigg|_{0}^{3} = 1 - e^{-3/4} = 0.5276. \)

Let \( Y \) be the number of days a person is served in less than 3 minutes. Then

\[
P(Y \geq 4) = \sum_{x=1}^{6} b(y; 6, 1 - e^{-3/4}) = \sum_{x=1}^{6} \binom{6}{x} (0.5276)^x (0.4724)^{6-x} = 0.3968. \]

6.47 (a) \( E(X) = \int_{0}^{\infty} x^2 e^{-x^2/2} \, dx = \int_{0}^{\infty} -xe^{-x^2/2} \, dx + \int_{0}^{\infty} e^{-x^2/2} \, dx = 0 + \sqrt{2\pi} \cdot \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-x^2/2} \, dx = \sqrt{\frac{2\pi}{\pi}} = 1.2533. \)

(b) \( P(X > 2) = \int_{0}^{\infty} xe^{-x^2/2} \, dx = -e^{-x^2/2} \bigg|_{2}^{\infty} = e^{-2} = 0.1353. \)

6.49 The density function is \( f(x) = 3(1-x)^2, \) for \( 0 < x < 1. \)

(a) \( \mu = \frac{1}{1+3} = \frac{1}{4}. \) To compute median, notice the c.d.f. is \( F(x) = 1 - (1-x)^3, \) for \( 0 < x < 1. \)

Hence, solving \( 1 - (1-m)^3 = \frac{1}{2} \) we obtain \( m = 0.206. \)

(b) \( \sigma^2 = \frac{(1/3)^2}{(1+3)^2(1+3+1)} = 0.0375. \)

(c) \( P(X > \frac{1}{3}) = (1 - 1/3)^3 = 0.2963. \)

6.56 \( P(X > 270) = 1 - \Phi \left( \frac{\ln 270 - 4}{2} \right) = 1 - \Phi(0.7992) = 0.2119. \)

7.1 From \( y = 2x - 1 \) we obtain \( x = (y + 1)/2, \) and given \( x = 1, 2, \) and \( 3, \) then

\[ g(y) = f[(y+1)/2] = 1/3, \quad \text{for } y = 1, 3, 5. \]

7.2 From \( y = x^2, \) \( x = 0, 1, 2, 3, \) we obtain \( x = \sqrt{y}, \)

\[ g(y) = f(\sqrt{y}) = \left( \frac{3}{\sqrt{y}} \right) \left( \frac{2}{5} \right)^{\sqrt{y}} \left( \frac{3}{5} \right)^{3-\sqrt{y}}, \quad \text{for } y = 0, 1, 4, 9. \]

7.5 The inverse function of \( y = -2\ln x \) is given by \( x = e^{-y/2} \) from which we obtain \( |J| = \left| -e^{-y/2}/2 \right| = e^{-y/2}/2. \) Now,

\[ g(y) = f(e^{y/2}) |J| = e^{-y/2}/2, \quad y > 0, \]

which is a chi-squared distribution with 2 degrees of freedom.