1. (20 points) A particular brand of diet margarine was analyzed to determine the level of polyunsaturated fatty acid (in percentages). A sample of six packages resulted in the following data:

16.8  17.2  17.4  16.9  16.6  17.1

Assume measurements of the polyunsaturated fatty acid level are normally distributed.
Find a 95% confidence interval for the mean percentage level of polyunsaturated fatty acid.

\[
\bar{x} = \frac{1}{6} \left( 16.8 + 17.2 + 17.4 + 16.9 + 16.6 + 17.1 \right) = 17.0
\]

\[
\sigma^2 = \frac{1}{5} \left( (-0.2)^2 + (2)^2 + (0.4)^2 + (-1)^2 + (-4)^2 + (-1)^2 \right) = \frac{0.42}{5} = 0.084
\]

\[
\sigma = \sqrt{0.084} = 0.2898
\]

\[
\frac{\sigma}{\sqrt{n}} = \frac{0.2898}{\sqrt{6}} = 0.118
\]

\[
t = \frac{x}{\sigma / \sqrt{n}} = \frac{5}{0.0825} \approx 5.971
\]

\[
17 \pm 5.971 \times 0.118
\]

17 \pm 0.3042

16.70 to 17.30
2. (20 points) The melting points of two alloys used in formulating solder were investigated by melting 21 samples of each material. The sample mean and standard deviation for alloy 1 were $\bar{x}_1 = 420^\circ$F and $s_1 = 4^\circ$F, while for alloy 2 they were $\bar{x}_2 = 426^\circ$F and $s_2 = 3^\circ$F. An engineer claims that the two alloys actually do have the same melting point. Test this claim, using $\alpha = 0.05$. Assume that both populations are normally distributed and have the same variance.

$$H_0: \mu_1 = \mu_2 \quad \text{vs.} \quad H_1: \mu_1 \neq \mu_2$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{d_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $d_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}$

reject $H_0$ if $|t| > t_{\alpha/2, n_1+n_2-2} = t_{0.025, 40} = 2.021$

$$d_p = \sqrt{\frac{20 \cdot 4^2 + 20 \cdot 3^2}{40}} = 3.5355$$

$$t = \frac{420 - 426}{3.5355 \sqrt{\frac{1}{21} + \frac{1}{21}}} = -5.4991$$

$|t| = 5.4991 > 2.021$ so we reject $H_0$,

and conclude that the alloys' melting points are different.
3. (20 points) The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. A sample of 10 tubes results in \( \bar{x} = 317.2 \mu A \) and \( s = 15.7 \mu A \). (Assume that current measurements are normally distributed.)

(a) Find a 95% confidence interval for the mean current required to achieve that level of brightness.

\[
\bar{x} \pm t_{0.025, n-1} \frac{s}{\sqrt{n}}
\]

\[
t_{0.025, 9} = 2.262
\]

\[
317.2 \pm (2.262) \frac{15.7}{\sqrt{10}}
\]

\[
317.2 \pm 11.2303
\]

\[
305.97 \text{ to } 328.43
\]

3 (a) \( 306.0 \text{ to } 328.4 \)

(b) Find a 95% prediction interval for the current required to achieve that level of brightness for a randomly-selected tube.

\[
\bar{x} \pm t_{0.025, n-1} \sqrt{\frac{s^2}{n} + \frac{(X - \bar{x})^2}{n}}
\]

\[
317.2 \pm (2.262)(15.7) \sqrt{1 + \frac{1}{10}}
\]

\[
317.2 \pm 37.2468
\]

\[
279.95 \text{ to } 354.4468
\]

3 (b) \( 280.0 \text{ to } 354.4 \)
4. (20 points) A random sample of 500 adult residents of Hennepin County found that 385 were in favor of increasing the highway speed limit to 75 mph, while another sample of 400 adult residents of Ramsey County found that 267 were in favor of the increased speed limit. Do these data indicate that there is a difference between the adult residents of the two counties in the support for increasing the speed limit? Use $\alpha = 0.05$.

$$H_0: \hat{p}_1 = \hat{p}_2 \quad \text{vs} \quad H_1: \hat{p}_1 \neq \hat{p}_2$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{385 + 267}{500 + 400} = \frac{652}{900} = 0.7244$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\frac{385}{500} - \frac{267}{400}}{\sqrt{(0.7244)(0.2756)\left(\frac{1}{500} + \frac{1}{400}\right)}}$$

$$= 3.4199 > 1.96 = Z_{0.02} \text{ so reject } H_0,$$

and conclude that there is a difference in the support for increasing the speed limit between adult residents in these counties.
5. (20 points) The Economist collects data each year on the price of a Big Mac in various countries around the world. The price of a Big Mac for a random sample of McDonalds restaurants in Europe in May 2009 resulted in the following Big Mac prices (after conversion to U.S. dollars):

3.80  5.89  4.92  3.88  2.65  5.57  6.39  3.24

(The mean and standard deviation of this sample are 4.542 and 1.347.) The mean price of a Big Mac in the U.S. in May 2009 was $3.57. Does this sample provide convincing evidence that the mean price of a Big Mac in Europe is greater than the reported U.S. price? Assume the distribution of Big Mac prices in Europe is normal, and test an appropriate hypothesis with $\alpha = 0.05$.

\[
H_0: \mu = 3.57 \quad \text{vs} \quad H_1: \mu > 3.57
\]

\[
t = \frac{\bar{x} - 3.57}{s/\sqrt{n}}
\]

(reject $H_0$ if $t > t_{.05, n-1}$)

\[
t_{.05, 7} = 1.895
\]

\[
t = \frac{4.542 - 3.57}{1.347/\sqrt{8}} = 2.041 > 1.895
\]

so reject $H_0$ and conclude that Big Macs do cost more (on average) in Europe than in the U.S.