

**Gambling, Probability, and Statistics:  
a biased, anecdotal, historical account**

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**X**

X, the professor, always dreads  
Cocktail party gatherings,  
Having to divulge he teaches Statistics.

The nose of his interrogator usually wriggles,  
Memories bubble in her green margarita.  
A proxy here to regale  
and even the score for old tortures.

J. Mills Dickey (1939-2008), statistician and poet

## **History of Gambling: a Few Approximate Dates**

3000 BCE in Mesopotamia: earliest known six-sided dice

3000 BCE in Egypt: anti-gambling laws

2000 BCE in China: keno played

1500 BCE in India: dice game in the Mahabharata

30 CE: casting of lots for Christ's clothes in the Bible

**Probability theory and mathematical statistics come much later**

**Cardano: the Gambling Scholar 1501-1576**  
**Book on Games of Chance: 1st published 1663**  
Dover edition 2015



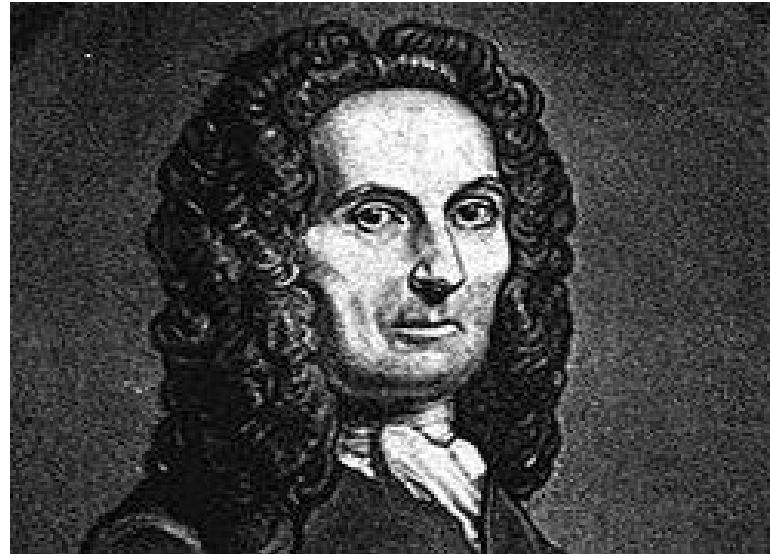
## Fermat-Pascal Correspondence 1654 Division of the Stakes



**Jacob Bernoulli 1654-1705**  
**Law of Large Numbers**



**Abraham de Moivre 1667-1754**  
**Central Limit Theorem**



**Louis Bachelier 1870-1946 (“father of math finance”)  
Théorie de la Speculation 1900**

“It is almost always gambling that enables one to form a fairly clear idea of a manifestation of chance ... it is, therefore, gambling that one must strive to understand, but one should understand it in a philosophic sense, free from all vulgar ideas.”



## **The Gambler's Ruin (Annals of Math. 1909)**

**“Bold Play” is optimal?**

**J. L. Coolidge 1873-1954**

“The player's best chance of winning a certain sum at a disadvantageous game, is to stake the sum which will bring him that return in one play, or, if that be not allowed, to make always the largest stake which the banker will accept.”

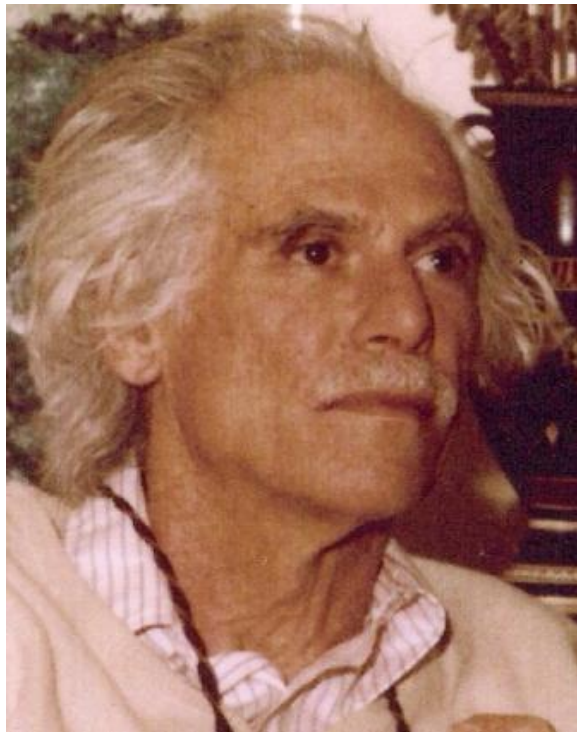
## The Game of Red-and-Black

You have some amount of money, say \$200, and are in desperate need to have a larger amount, say \$500. Your only opportunity is to make bets on Red (or Black) at a roulette wheel. So you win each bet with probability  $18/38$  and lose with probability  $20/38$ .

Bold play is to bet all of your \$200. If you lose, the game is over. If you win, you have \$400, and should then bet \$100. And so on, until you reach your goal or go bankrupt.

If there is a house limit, say \$100, you then begin by betting the limit. And so on.

**How to Gamble If You Must:  
Inequalities for Stochastic Processes 1965  
Lester Dubins (1920-2010), Leonard Savage (1917-1971)**



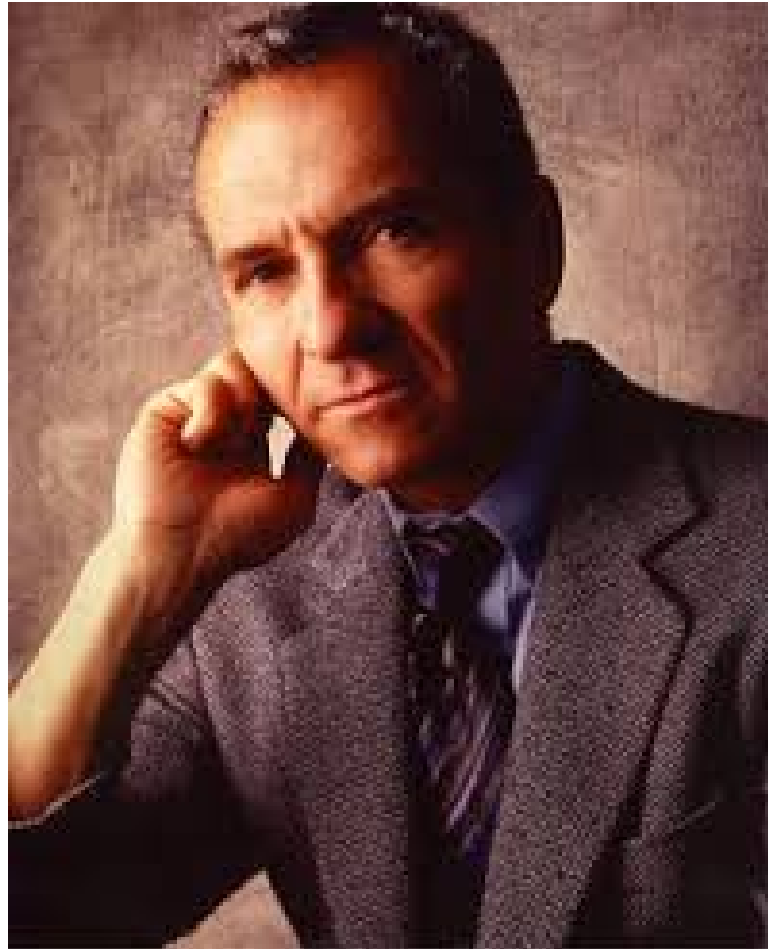
“We have not seen how to rescue much from Coolidge’s faulty proof.”

**Red-and-Black with a house limit  
David Heath 1943-2011**



Bold play is optimal for red-and-black if the goal is \$500, and the house limit is \$100 but not if the limit is \$200.

**Edward O. Thorp: Another Gambling Scholar 1932-  
Beat the Dealer 1962, Beat the Market 1967**



## Meaning of Probability?

**Symmetric outcomes:**  $\frac{\text{number of favorable outcomes}}{\text{total number}}$

The probability of getting a 6 on a fair die =  $\frac{1}{6}$ .

**Longrun frequency:**  $\frac{\text{number of successes in many trials}}{\text{number of trials}}$

The probability of getting a 6 on a fair die is about

$$\frac{\text{number of 6's in } n \text{ tosses}}{n}$$

for  $n$  large.

Probability Vikings win their next game = ?

## Truth and Probability Frank P. Ramsey 1903-1930



“In this essay the Theory of Probability is taken as a branch of logic, the logic of partial belief and inconclusive argument ...”

## Subjective Probability and Coherent Odds

Bruno de Finetti 1906-1985

La prévision: ses lois logiques, ses sources subjectives

1937





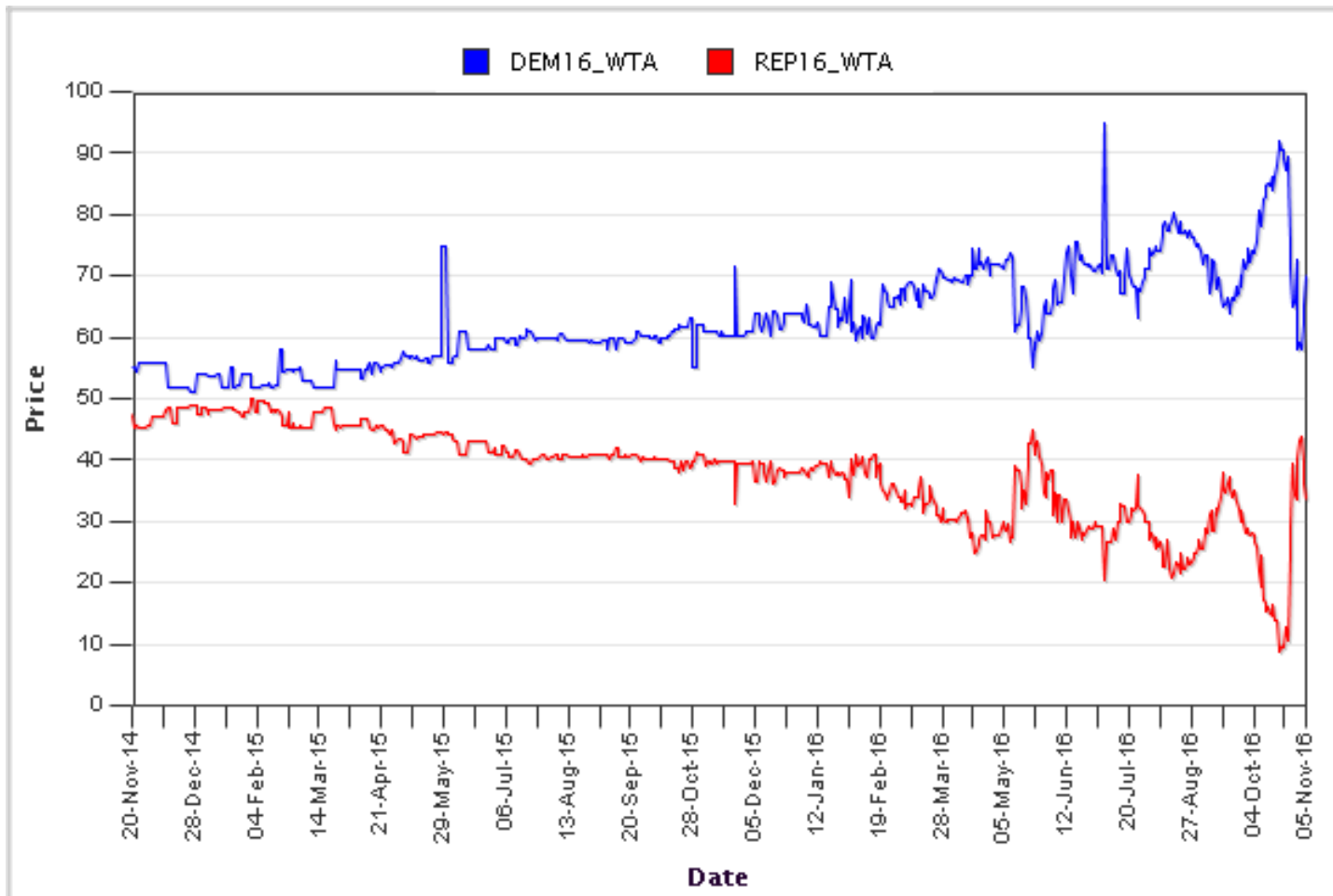
## **A Theorem of de Finetti**

If a bookie offers odds that are inconsistent with the laws of probability, then there is a way for a gambler to place bets that guarantee a winning outcome.

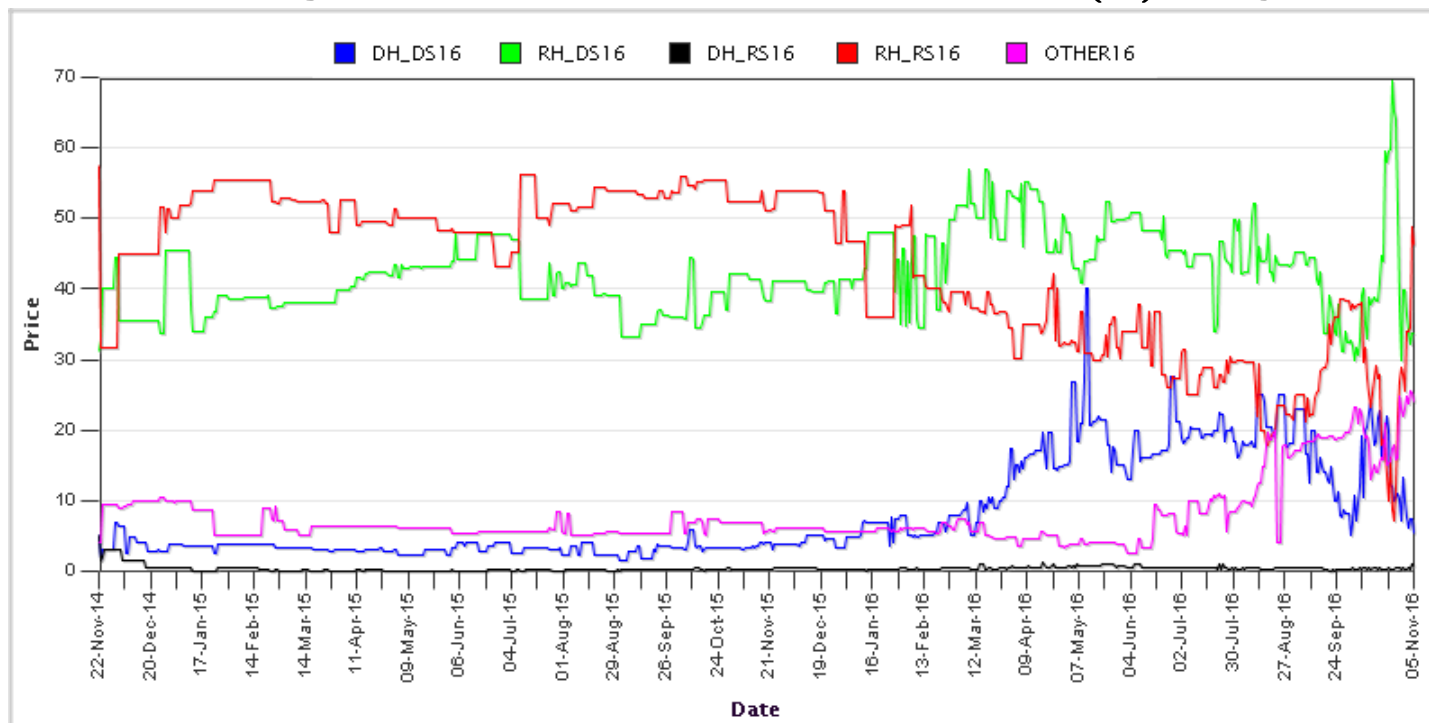
In the language of math finance: If prices are inconsistent with the laws of probability, then there is an arbitrage opportunity.

The Black-Scholes formula for pricing an option assumes there is no arbitrage opportunity.

# Iowa Election Market: Winner-Take-All



## Congressional Control Market (2).png



# Mathematical Statistics: Bayesians, Frequentists, and R. A. Fisher



**Thomas Bayes 1702-1761**

## Bayes' Theorem

The **posterior** probability that the hypothesis  $H$  is correct given the evidence  $E$  is related to the probability of  $E$  given that  $H$  is correct and the **prior** probability that  $H$  is correct:

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}.$$

Typically, neither the prior nor the posterior probability of  $H$  makes sense to a frequentist.

**Problem for Bayesians:** Where does the prior come from?

**Pierre-Simon Laplace 1749-1827**  
**An objective Bayesian in the 19th century**



**Principle of Insufficient Reason**

**R. A. Fisher 1890-1962: the greatest 20th century  
statistician**



Fisher believed Bayesian methods “must be wholly rejected.” He calculated “fiducial” probabilities for  $H$  given  $E$ .

**Jerzy Neyman (1894-1981), Egon Pearson (1895-1980)**  
**Two prominent frequentists not admired by Fisher**





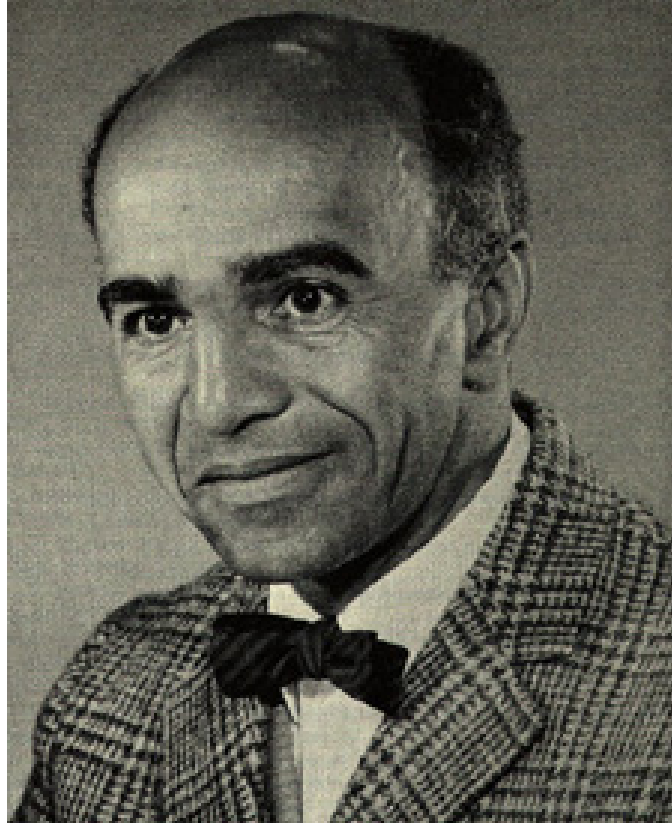
## **Leonard Savage again and the revival of the Bayesians**

### **The Foundations of Statistics 1954**

“... the theory of personal probability applied to statistics shows that many of the prominent frequentist devices can at best lead to accidental and approximate, not systematic and cogent, success ...”

**David Blackwell 1919-2010**

**Greatest 20th Century African-American Mathematician**



**Converted to a Bayesian by Savage**

**Written and Sung by David Blackwell, Altea, Spain 1987**

(Verse one)

Bayes!

You're the very best

Truer than all the rest

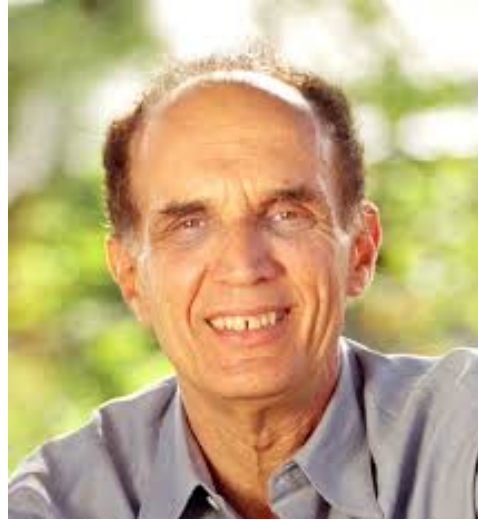
Confidence, Fiducial too

I don't need them

since I have you.

(To the tune of "Who stole my heart away.")

**Bradley Efron - born 1938 in St Paul**  
**Inventor of the bootstrap**



“Broadly speaking, Bayesian statistics dominated 19th Century statistical practice while the 20th Century was more frequentist. Whats going to happen in the 21st Century? One thing thats already happening is that scientists are bringing statisticians much bigger data sets to analyze, with millions of data points and thousands of parameters to consider all at once.”

**Final Anecdotes on Statistics, the Law, and Data**  
**The Rajender Decree 1980-1991**



**Charlotte Stiebel 1929-2014**  
**mathematician, feminist, lawyer**

## Pillsbury, Green Giant, and age discrimination (1983?)



Statistics, you see,  
is a wondrous cult  
For a non-mathematical mind,  
Which wants but the final,  
or end result -  
As to how it's attained  
is quite blind.

Anonymous

**Thanks for your attention!**