Stat 8311, Fall 2006, Power calculations

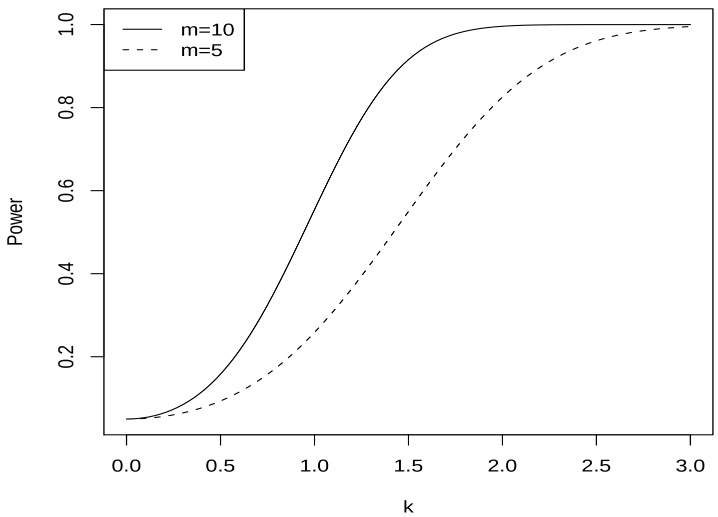
One-way design, t groups, m observations per group. The null hypothesis is $\mu in\mathrm{R}(J_t \otimes J_m)$, and the alternative is $\mu in\mathrm{R}(I_t \otimes J_m)$. Then the F test is $f = \parallel P_{E-E_o} y \parallel^2/(t-1)/\parallel Q_E y \parallel^2/(m(t-1)$, anf $f \sim F(t-1, m(t-1), \delta^2)$, with $\delta^2 = \parallel P_{E-E_0}\mu \parallel^2/\sigma^2 = m\sum(\gamma_i - \bar{\gamma})^2/\sigma^2$, where γ_i is the mean for group i.

In the first case, suppose that all the group means are equal, except in one group the mean is increased by $k\sigma$. Then $\delta^2 = (t-1)mk^2/t$. The calculations below are for $t = 5$, $m = 10$.

```
> qf(0.95, 4, 45)

[1] 2.578739

> pf(qf(0.95, 4, 45), 4, 45, 4 * 10 * c(1, 2, 3)^2/5,
+     lower.tail = FALSE)

[1] 0.5540384 0.9959826 0.9999999
```

To get a graph of power as a function of k, for fixed $m = 5, 10$:

```
> kvals <- seq(0, 3, length = 101)
> pow10 <- pf(qf(0.95, 4, 45), 4, 45, 4 * 10 * kvals^2/5,
+     lower.tail = FALSE)
> pow5 <- pf(qf(0.95, 4, 20), 4, 20, 4 * 5 * kvals^2/5,
+     lower.tail = FALSE)
> plot(kvals, pow10, type = "l", xlab = "k", ylab = "Power")
> lines(kvals, pow5, lty = 2)
> legend("topleft", legend = c("m=10", "m=5"), lty = 1:2)
```



In a second, case, we assume that in the alternative the γ_i can be ordered so that $|\gamma_{(i+1)} - \gamma_{(i)}| = k\sigma$. For this alternative, if $t = 5$, $\delta^2 = 10mk^2$. For $m = 5$, compare the power to the $m = 5$ case with the first alternative

```
> pow3 <- pf(qf(0.95, 4, 20), 4, 20, 10 * 5 * kvals^2,
+     lower.tail = FALSE)
```

```
> plot(kvals, pow3, type = "l", xlab = "k", ylab = "Power")
> lines(kvals, pow5, lty = 2)
> legend("bottomright", legend = c("Equal spacing", "One different"),
+     lty = 1:2)
```