

## Stat 8311 Fall 2006 – One factor anova

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> set.seed(121)
> mu <- c(2,2,3,3,5,5,5,0,0,0,0)
> (y <- mu + rnorm(11))
 [1] 1.74463953 2.10837472 3.12778056 2.91892655 4.28631628 6.61533208
 [7] 5.29726662 -0.05765318 -1.09960818 0.91354906 1.40279533
> (f <- rep(1:4,c(2,2,3,4)))
 [1] 1 1 2 2 3 3 3 4 4 4 4
> X <- t( matrix( c(1,1,0,0,0,0,0,0,0,0,0,0,
+                 0,0,1,1,0,0,0,0,0,0,0,0,
+                 0,0,0,0,1,1,1,0,0,0,0,0,
+                 0,0,0,0,0,0,0,1,1,1,1,1), byrow=T,ncol=11))
> QR <- qr(X) # QR decomposition
> Q<-qr.Q(QR) # Returns ONB for E
> (muhat <- qr.fitted(QR,y)) # Returns Q %>% t(Q) %>% y
 [1] 1.9265071 1.9265071 3.0233536 3.0233536 5.3996383 5.3996383 5.3996383
 [8] 0.2897708 0.2897708 0.2897708 0.2897708
> tapply(y,f,mean)
      1      2      3      4
1.9265071 3.0233536 5.3996383 0.2897708
> residuals <- y - muhat # same as qr.residuals(QR,y)
> (sigmahatsq <- sum(residuals^2)/(length(y)-length(unique(f))))
 [1] 0.9278342
> opt <- options(digits=2)
> sigmahatsq * Q %>% t(Q) # variance of muhat
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
 [1,] 0.46 0.46 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 [2,] 0.46 0.46 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 [3,] 0.00 0.00 0.46 0.46 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 [4,] 0.00 0.00 0.46 0.46 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 [5,] 0.00 0.00 0.00 0.00 0.31 0.31 0.31 0.00 0.00 0.00 0.00
 [6,] 0.00 0.00 0.00 0.00 0.31 0.31 0.31 0.00 0.00 0.00 0.00
 [7,] 0.00 0.00 0.00 0.00 0.31 0.31 0.31 0.00 0.00 0.00 0.00
 [8,] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.23 0.23 0.23 0.23
 [9,] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.23 0.23 0.23 0.23
[10,] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.23 0.23 0.23 0.23
[11,] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.23 0.23 0.23 0.23
> options(opt)
> sum(muhat^2) # same as t(y) %>% P y
 [1] 113.5083
> opt <- options(digits=2)
> (X1 <- model.matrix(~factor(f)))
      (Intercept) factor(f)2 factor(f)3 factor(f)4
1                1                0                0                0
2                1                0                0                0
3                1                1                0                0
4                1                1                0                0
5                1                0                1                0
6                1                0                1                0
7                1                0                1                0
8                1                0                0                1
9                1                0                0                1
10               1                0                0                1
11               1                0                0                1

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> QR1 <- qr(X1)
> (Q1 <- qr.Q(QR1))
      [,1] [,2]      [,3]      [,4]
[1,] -0.3 -0.14 -2.4e-01 -5.8e-01
[2,] -0.3 -0.14 -2.4e-01 -5.8e-01
[3,] -0.3  0.64  1.4e-17 -1.1e-16
[4,] -0.3  0.64  1.4e-17 -2.8e-17
[5,] -0.3 -0.14  4.7e-01 -2.8e-17
[6,] -0.3 -0.14  4.7e-01 -2.8e-17
[7,] -0.3 -0.14  4.7e-01 -2.8e-17
[8,] -0.3 -0.14 -2.4e-01  2.9e-01
[9,] -0.3 -0.14 -2.4e-01  2.9e-01
[10,] -0.3 -0.14 -2.4e-01  2.9e-01
[11,] -0.3 -0.14 -2.4e-01  2.9e-01
> range(Q %*% t(Q) - Q1 %*% t(Q1))
[1] -1.348796e-16  1.110223e-16
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> Jn <- rep(1,11)
> P1 <- Jn %*% t(Jn)/11
> Pstar <- (diag(Jn) - P1) %*% (Q %*% t(Q))
> f1 <-P1 %*% y
> f2 <-Pstar %*% y
> f3 <- f1 + f2
> cbind(f1, f2, f3, muhat)
              muhat
[1,] 2.5 -0.55 1.93  1.93
[2,] 2.5 -0.55 1.93  1.93
[3,] 2.5  0.55 3.02  3.02
[4,] 2.5  0.55 3.02  3.02
[5,] 2.5  2.92 5.40  5.40
[6,] 2.5  2.92 5.40  5.40
[7,] 2.5  2.92 5.40  5.40
[8,] 2.5 -2.19 0.29  0.29
[9,] 2.5 -2.19 0.29  0.29
[10,] 2.5 -2.19 0.29  0.29
[11,] 2.5 -2.19 0.29  0.29
> options(opt)

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