Assignment #2, Stat 8311, Fall 2006

Reading

Finish Chapter 2.

Reminder

No class on Monday, October 2.

Problems

Due: Monday, September 25, 2006

1. Consider the following linear transformations of $\left(\begin{smallmatrix}\eta_1\\ \eta_2\\ \eta_3\end{smallmatrix}\right) \in \Re^3$. In each case, show whether it is or is not a projection. When it is a projection, what is it on and what is it along?

   (a) $A\begin{pmatrix}\eta_1\\ \eta_2\\ \eta_3\end{pmatrix} = \begin{pmatrix}\eta_1-\eta_3\\ \eta_2-\eta_3\\ 0\end{pmatrix}$

   (b) $B\begin{pmatrix}\eta_1\\ \eta_2\\ \eta_3\end{pmatrix} = \begin{pmatrix}\eta_1\\ \eta_1+\eta_2\\ \eta_1+\eta_2+\eta_3\end{pmatrix}$

2. For each of the following functions of two arguments in $\Re^2$, show whether or not it is an inner product.

   (a) $x_1y_1 - x_2y_2$

   (b) $2x_1y_1 + x_2y_1 + x_1x_2 + 3x_2y_2$

   (c) $3x_1y_1 + 5x_2y_2$

   (d) $2x_1y_1 + x_1y_2 + 3x_2y_2$

3. Show that for any inner product, and the Euclidean norm $\| x \|^2 = (x, x)$,

$$\| x + y \|^2 + \| x - y \|^2 = 2(\| x \|^2 + \| y \|^2)$$

$$(x, y) = \frac{1}{4}(\| x + y \|^2 - \| x - y \|^2).$$

   Thus if we know lengths, we know the inner product. Does this relationship hold for other norms?

4. Consider the linear model with response $y \in \Re^3$, and $\mathrm{E}(y) = \mu$, with $\mu \in C$, with

$$C = \mathrm{R}\left(\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right)$$

Define:

$$D = \{y|(x, y) = 0, x \in C\},$$

where (x, y) is the usual inner product, and

$$E = \{x|x = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \alpha \in \Re\}.$$

Verify that $C + D = C + E = \Re^3$ and $C \cap D = C \cap E = \{0\}$. Find the projections $P_{C|D}$, and $P_{C|E}$.

5. Consider $\mathcal{P}_3$, the vector space of polynomials of degree ≤ 3. We know a basis for $\mathcal{P}_3$ is $\{1, t, t^2, t^3\}$. Also, $\int_0^1 p(t)q(t)dt$ is an inner product on $\mathcal{P}_3$. Apply the Gram-Schmidt method to find an orthonormal basis for $\mathcal{P}_3$.

6. Let $y_1, \ldots, y_n$ be random variables with var$(y_i) = \sigma_{ii} > 0$, cov$(y_i, y_j) = \sigma_{ij}$. Find new random variables $z_1, \ldots, z_n$ as linear combinations of the $y_i$so that: var$(z_j) = 1$ and cov$(z_i, z_j) = 0, i \neq j)$. (Hint: Apply the Gram-Schmidt method to random variables rather than vectors. Give explicit formulas for z_1 and z_2, and a general formula for z_j, $j > 2$.)