STAT4102 Homework 1

Due January 30 in class. (You can also turn in to Xiaoqiao Wei's mailbox.)

Simulation 1: What does the sampling distribution of the sample mean look like if samples are taken from an approximately normal distribution? Use the applet *Basic* found at `http://www.thomsonedu.com/statistics/book_content/0495110817_wackerly/applets/seeingstats/Chpt7/index.html` (direct link on the homework page) to complete the following. The population to be sampled is approximately normal with $\mu = 16.5$ and $\sigma = 6.03$ (these values are given above the population histogram and denoted M and S, respectively.

1. Use the button "Next Obs" to select a single value from the approximately normal population. Click the button four more times to complete a sample of size 5. What value did you obtain for the mean of this sample? Locate this value on the bottom histogram (the histogram for the values of $\overline{Y}$).

2. Click the button "Reset" to clear the middle graph. Click the button "Next Obs" five more times to obtain another sample of size 5 from the population. What value did you obtain for this new sample? Is the value that you obtained equal to the value you obtained in part (a)? Why or why not?

3. Use the button "1 Sample" eight more times to obtain a total of ten values of the sample mean. Look at the histogram of these ten means.

   (a) What do you observe?
   (b) How does the mean of these 10 $\bar{y}$-values compare to the population mean μ?

4. Use the button "1 Sample" until you have obtained and plotted 25 realized values for the sample mean $\overline{Y}$, each based on a sample size of size 5.

   (a) What do you observe about the shape of the histogram of the 25 values of $\bar{y}_i$, $i = 1, 2, \ldots, 25$?
   (b) How does the value of the standard deviation of the 25 $\bar{y}$ values compare with the theoretical value of $\sigma_{\overline{Y}}$ obtained in Example 5.27 where we showed that, if $\overline{Y}$ is computed based on a sample size of size n, then $V(\overline{Y}) = \sigma^2/n$?

5. Click the button "1000 Samples" a few times, observing changes to the histogram as you generate more and more realized values of the sample mean. What do you observe about the shape of the resulting histogram for the simulated sampling distribution of $\overline{Y}$?

6. Click the button "Toggle Normal" to overlay (in green) the normal distribution with the same mean and standard deviation as the set of values of $\overline{Y}$ that you previously generated. Does this normal distribution appear to be a good approximation to the sampling distribution of $\overline{Y}$?

   This simulation is problem 7.5 in the seventh edition.

Simulation 2: Suppose that the population of interest does not have a normal distribution. What does the sampling distribution of $\overline{Y}$ look like, and what is the effect of the sample size on the sampling distribution of $\overline{Y}$? Use the applet *Sample Size* (direct link on the homework page) to complete the following.

1. Use the up/down arrow to the left of the histogram of the population distribution to select the "Skewed" distribution. What is the mean and standard deviation of this population, from which samples will be selected? (These values are labeled M and S and are given above the population histogram.)

2. Use the up/down arrows in the left and right "Sample Size" boxes to select samples of size 1 and 3. Click the button "1 Sample" a few times. What is similar about the two histograms that you generated? What is different about them?

3. Click the button "1000 Samples" a few times. Now, what is similar and what is different about the two histograms? Do the generated histograms have the shapes that you expected? Why or why not?

4. Are the means and standard deviations of the two sampling distributions close to the values that you expected? (Remember that $V(\overline{Y}) = \sigma^2/n$.)

5. Click the button "Toggle Normal." What do you observe about the adequacy of the approximating normal distributions?

6. Click on the two generated sampling distributions to pop up windows for each. Go back to the main window, and use the up/down arrows in the left and right "Sample Size" boxes to select samples of size 10 and 25. Click the button "1000 Samples" a few times, and click the button "Toggle Normal." You now have graphs of the sample means based on samples of size 1, 3, 10, and 25. What do you observe about the adequacy of the normal approximation as the sample size increases?

7. Recreate those four graphs, but using the "U-Shaped" distribution instead of "Skewed." What differences do you note?

This simulation is problems 7.40 and 7.41 in the seventh edition.

Additional Problems: Chapter 7: 23, 24, 38, 40, 74
These are 43, 44, 58, 60, and 100 in the seventh edition.