Review

- More on Estimation
  - Finding estimators using Maximum Likelihood and the Method of Moments
  - Large sample properties of MLEs (won't be on exam)
- Concepts of Statistical Testing
  - Elements of a test (null and alternate hypotheses, test statistic, rejection region)
  - Type I and Type II Errors
  - Power
  - relationship of these to sample size
  - p-values
- Large Sample Testing
  - execute tests for the four basic large sample cases
  - find Type II Error/Power for a given sample size and vice versa
  - find the p-value for a given test
  - be able to relate hypothesis testing, p-values, and confidence intervals
- Theory of Hypothesis Testing
  - Use Neyman Pearson Lemma to find the most powerful test for simple hypotheses
  - Extend the Neyman Pearson Lemma to find the UMP for composite hypotheses, or show that a UMP test does not exist.
  - Test composite hypotheses using the Likelihood Ratio Test

Exam Logistics

- You are permitted two sheets of notes.
- Bring a calculator.
- I'll give you copies of the front and back covers of the text and any other tables (such as the χ^2) that you may need.

Practice Exam

1. Suppose you have a data set of 50 observations with sample mean $\bar{x} = 12$ and sample variance $s^2 = 56 = 7.5^2$.
   (a) Find a 95% confidence interval for the population mean μ. Does this confidence interval support the hypothesis that $\mu = 10$, or not? Explain why.
   (b) Find a p-value for testing $\mu = 10$ against $\mu \neq 10$. Give an interpretation of this p-value in your own words.
   (c) Do the confidence interval and the p-value lead you to the same conclusion? Did you expect this? Explain why or why not.

2. Consider this same data set of 50 observations. Now, you wish to test the null hypothesis of $\mu = 10$ against the alternate hypothesis of $\mu > 10$.
   (a) Against $\mu = 12$, you want the power to be 0.8. Find the rejection region.
   (b) What's the size of the Type I error for this rejection region?
   (c) Using this rejection region, how large of a sample size would you need to reduce the Type I error to 0.05? Does this increase or decrease the power? Explain.

3. Let $y_1, \ldots, y_n$ be a random sample of size n from a Gamma distribution with $\alpha = 2$ and $\beta = 1/\theta$, so

$$f(y|\theta) = \theta^2 y e^{-\theta y} \quad \text{for } y > 0,$$

   where $\theta > 0$.
   (a) Derive the maximum likelihood and the method of moments estimator for θ.
   (b) Use the Neyman Pearson Lemma to find the rejection region for $H_0 : \theta = 1$ vs. $H_a : \theta = 2$.
   (c) Is this a uniformly most powerful test for testing against $H_a : \theta > 1$? Explain why or why not.
   (d) Perform the test, using $n = 10$ and $\bar{y} = 1.5$. Use the fact that $(2\theta) \sum_{i=1}^{n} y_i \sim \chi^2(4n)$.

4. We have independent samples of size 20 each from three independent exponential populations, with means β_1, β_2, and β_3. The sample means are 3, 4, and 8. Use the likelihood ratio test to test the null hypothesis that $\beta_1 = \beta_2 = \beta_3$ against the alternate that at least two are different.