Nonparametric significance testing

The Z test requires large sample sizes, so the CLT gives us normality. The T test requires near normality. What do we do when we don't have either? These methods are called nonparametric methods because they do not assume that the data follows any particular parametric family. They follow the same steps we used for the Z and T significance tests.

1 Sign Test

The sign test tests the median, not the mean. This is because no matter what the distribution is, it is equally likely for any data point to be above or below the median (called m). So for our null hypothesis, we assume the median is equal to some value m_0. We then count the number of data points above (or below) m_0, and calculate a p-value based on the probability of getting that number of data points (or more) above (or below) m_0. The steps are:

1. $H_0 : m = m_0$.
   Determine what m_0 is from the context of the problem. Often these problems are about differences; if you're looking for evidence of a difference, $m_0 = 0$.

2. $H_A : m > m_0$, or $m < m_0$, or $m \neq m_0$, depending on context.
   This determines if the test is one or two-tailed.

3. The test statistic, called Y, is the number of values above m_0. (You can also use the number below; that switches the direction of the inequalities that follow.) Since any value is equally likely to be above or below, the distribution of this statistic is $Y \sim Bin(n, 0.5)$.

4. What the extreme values are depends on H_A. For example, under $H_A : m < m_0$, we would expect an unusually small number to be greater than m_0.

5. Calculating the p-value: If we actually observe y values above m_0, for $H_A : m < m_0$, we need to find $P(Y < y)$.

   For small values of n we can do this using the cumulative table for the Binomial, Table Ib (p. 650); for large values of n we can use the CLT to approximate the binomial with a normal distribution.

   For a one-tailed test, you simply need the probability from Table Ib. For a two-tailed test you'd have to double this number.

Example 10.5a is an example of this with numbers.

2 Signed-Rank Test

The signed rank test is not quite as general as the sign test. It requires a population that has a symmetric density function, because then a given data point is not only equally likely to above or below the median, but has the same distribution above or below the median.

1. $H_0 : m = m_0$

2. $H_A : m > m_0$, or $m < m_0$, or $m \neq m_0$, depending on context.

3. To find the test statistic, subtract m_0 from the data points, rank these numbers by absolute value, and add the sum of the ranks of the negative (or positive numbers). To use the table ($n \leq 15$), you'll need the smaller of the two.

   For example, here's the data from Example 10.5b, ranked: (In this example $m_0 = 0$.)

Rank	1	2	3	4	5	6	7	8	9
Data	$+1$	-3	-3	$+5$	-6	-7	-7	-9	-17

   The ranks of the negative numbers are $2, 3, 5, 6, 7, 8, 9$, which adds to 40. The ranks of the positive numbers are 1 and 4, which add to 5. So we use the ranks of the positive numbers because we'll need to use the table, as $n = 9 < 15$.

   This test statistic has an interesting distribution. You can read about it on p. 440-1 if you like. For $n \leq 15$, the (one-sided) tail probabilities are in Table VI (p. 664), for $n > 15$, its approximately normal with mean $n(n+1)/4$ and variance $n(n+1)(2n+1)/24$.

4. Extreme values will depend on the alternate hypothesis and whether you're using the sum of the ranks of the positive or negative values.

   For this example, $H_A : m_0 < 0$, so extreme values means few positive values, so we need the tail for a rank-sum of 5 or less.

5. Since $n = 11$, we use Table VI to find the p-value; in this table n is the sample size and c is the rank-sum; 9 and 5 respectively in this case. This combination gives a p-value of 0.02.

   For a two-sided test, you'd have to double this value.