While these solutions are accurate and have all the final results, they should not be considered complete; some steps, explanations, and wordings have been left out. But it should be enough to help you determine if you've gotten the right answer or not. More details can be requested over email.

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1) This problem can be done either with comparing two proportions or by a test for homogeneity using goodness of fit, using using the chi-squared test or the likelihood ratio test.

For comparing two populations, $\hat{p}_1 = 0.424$, $\hat{p}_2 = 0.540$, and $\hat{p} = 0.458$. Then the test statistic is

$$\frac{\hat{p}_2 - \hat{p}_1}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}} = \frac{0.116}{0.019} = 6.105$$

and the p-value is $P(|Z| > 6.105) \approx 0$.

For either of the goodness of fit tests, we find the expected values under the null:

	men	women
grad	1068	439
not	1264	520

Then both the chi-squared test statistic and $-2\log(\Lambda)$ are about 37.
The p-value is $P(\chi_1^2 > 37) \approx 0$.

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2) Let $x_1, \ldots, x_5$ be the five data points. Then the likelihood of *theta* is

$$L(\theta|\boldsymbol{x}) = \theta^{10} \left(\prod x_i\right) e^{-\theta \sum x_i}$$

Maximizing this, the MLE for θ is $2/\bar{x} = 0.8547$. Then the likelihood ratio for testing $\theta = 0.25$ against $\theta \neq 0.25$ is

$$\Lambda = \frac{(0.25)^{10} e^{-0.25(11.7)}}{(0.8547)^{10} e^{-0.8547(11.7)}} = 0.0054187$$

and $-2\log\Lambda = 10.44$, so the p-value is $P(\chi_4^2 > 10.44) = 0.034$.

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3) The true ANOVA table is

	Df	Sum Sq	Mean Sq	F value
Treatment	4	11535760	2883940	6.43
Error	59	26448144	448273.6	
Total	63	37983904		

Then the p-value is $P(F_{4,59} > 6.43) = 0.00023$.

4) For simple hypotheses, the most powerful test is to reject the null when $f_0(x)/f_A(x) < K$, which is when $1/2x < K$, or $x > 1/2k$; let $k' = 1/2k$ so we reject when $x > k'$. We then need to find the value of k' so that $\alpha = 0.1$. That means that the probability of rejecting when H_0 is true (that is, $P(X > k'|f_0)$) needs to be 0.1. Under H_0 the density of X is 1 for $0 < X < 1$, so $P(X > 0.9) = 0.1$, and $k' = 0.9$; thus we reject the null when $X > 0.9$.

    The power of this test is the probability of rejection when H_A is true (that is, when $P(X > 0.9|f_A)$) and the density is $2x$ for $0 < x < 1$; that works out to be 0.19. (It may help to draw the picture and find the probability by geometry.)

    Finally, if $X = 0.7$, this is not in our rejection region so we accept H_0.

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5) $H_0 : \mu = 1520$, $H_A : \mu \leq 1520$. The p-value is

$$P(\bar{x} < 1498.3) = P(Z < (1498.3 - 1520)/(85/\sqrt{40})$$
$$= P(Z < -1.614) = 0.053.$$

    At the 0.01 significance level, we'll reject when the p-value is less than 0.01; from the table $P(Z < -2.33) = 0.01$, so we reject when $(\bar{x} - 1520)/(85\sqrt{40}) < -2.33$, or $\bar{x} < 1488.68$. Then the power for a given value of μ is $P(\bar{x} < 1488.7|\mu)$; when $\mu = 1488.7$, this will be 0.5 (the desired power). So it can detect a difference of $1520 - 1488.7 = 31.3$.

    To perform the sign test, let Y be the number of observations less than 40; then under the null that the median is 20, $Y \sim \text{Bin}(40, 0.5)$. Using this, the p-value is the probability that $Y > 30$. Doing a normal approximation,

$$P(Y > 30) = P(Z > (30 - 40(0.5))/\sqrt{40(0.5)(1 - 0.5)}$$
$$= P(Z > 3.16) \sim 0.0008.$$

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6) The density of a Bernoulli is $f(x) = p^x(1-p)^{1-x}$ for x either 0 or 1. Then the likelihood for a sample of size n, letting $x_1, \ldots, x_n$ be the sample points, is $p^{\sum x_i}(1-p)^{n-\sum x_i}$. Maximizing this gives $\hat{p} = \sum x_i/n$.

    For the method of moments, we use that the expected value of a Bernoulli is p; setting this equal to the sample mean $\bar{x}$, we get $p_{MOM} = \bar{x}$.

    For the Bayesian method, the prior is proportional to $p^{\alpha-1}(1-p)^{\beta-1}$. Then the posterior is proportional to the prior times the likelihood, which is $p^{\bar{x}+\alpha-1}(1-p)^{n-\bar{x}+\beta-1}$. We recognizing this as a Beta distribution with mean $(\alpha+\bar{x})/(\alpha+\beta+n)$; that's the Bayesian estimate of p.