STAT4102 Exam 1 Makeup, Spring 2007

NAME:____

This makeup portion is a take-home exam. It should be your work only. You may use your text and other written resources, but you may not interact with any other person about the specific material here, except for the instructor and the TA.

Please sign your name below agreeing to this:

Signature:____

There are two options for this makeup portion.

Option 1: Replace your score on up to **three of the last four** problems.
You may choose up to three of the last four problems to be replaced. Simply do the corresponding problems. There is no need to attach your inclass exam.

Option 2: Replace your score on the multiple choice and **one** of the last four problems.
For the multiple choice questions you got wrong (numbers 1-10), explain why the correct answer is correct, and the wrong answers are wrong. Then choose one of the last four problems to replace and do the corresponding problem. For this option, attach your in-class exam so I know which you got wrong.

This will never hurt your score. For each problem, I will take the higher of your inclass score and the new score.

To replace 11:

a. (2 pts) Explain the CLT in your own words.

b. (2 pts) A student wishes to survey fellow students about their yearly income. Speculate about what the population distribution looks like. Specifically, is it approximately normally distributed? Why or why not?

c. (2 pts) This student randomly samples 200 fellow students and gets a sample mean of \$8,000 and a standard deviation of \$3000. What is the approximate sampling distribution of the sample mean? If you can, state the approximate mean and variance, as well as some description of its shape.

d. (2 pts) Find the approximate probability that the sample mean is greater than \$8200.

e. (2pts) What sample size would you need to make this probability approximately equal to 0.05? Your answer should be an integer, so be sure to round correctly.

To replace 12:

a. (2pts) Your instructor likes to say that the likelihood is proportional to the joint density, but that the roles of the parameter and the data are reversed. Explain what he means when he says the roles are reversed.

Now consider a random sample of size n from a distribution with density

$$f(x|\beta) = 3\beta x^2 \exp(-\beta x^3) \quad \text{for } x \geq 0,$$

where β must be positive. For this distribution, $EX \approx \frac{9}{10}\beta^{-1/3}$.

b. (2 pts) What's the likelihood function for β?

c. (2 pts) Find a one-dimensional sufficient statistic for β.

d. (2 pts) Calculate the maximum likelihood estimator of β.
(Don't bother checking the second derivative.)

e. (2 pts) Calculate an approximate method of moments estimator of β.

To replace 13:

A curious statistician wants to find the probability p that a Hershey's chocolate kiss lands on its base when tossed onto a surface. She has reason to believe that this probability is related to the ratio of base surface area to total surface area, so she decides to do a Bayesian analysis using a Beta$(3, 6)$ as a prior **(different prior than in-class)**, and bases her estimates on squared error loss.

However, she does a **different experiment** than the one described in the in-class portion. She takes 50 kisses. For each one, she repeatedly tosses it until it lands on its base, and records the number of times she tosses it (so it's Geometric). The average number of times she has to toss a kiss is 3.2.

Remember that for $X \sim \text{Beta}(\alpha, \beta)$, the density is

$$f(x|\alpha, \beta) \propto x^{\alpha-1}(1-x)^{\beta-1} \quad \text{for} \quad 0 < x < 1,$$

and $EX = \alpha/(\alpha + \beta)$, and for $Y \sim \text{Geo}(p)$, the probability function is

$$P(Y = y) = p(1-p)^y \quad \text{for } y = 1, 2, 3, \ldots$$

and $EY = 1/p$ and $\operatorname{Var} Y = (1-p)/p^2$.

a. (4 pts) Find her prior estimate of p.

b. (3 pts) Find the posterior distribution of p. Show your work.

c. (3 pts) Find the posterior estimate of p.

To replace 14:

a. (4 pts) For any distribution with finite variance, show that the sample mean $\bar{x}$ is consistent for the population mean μ.

b. (2 pts) Explain what the Cramer-Rao Lower Bound is a lower bound of.

c. (4 pts) For a random sample of size n from a Poi(λ) distribution (see p. 142 for definition), determine if $\bar{x}$ is efficient by calculating the Cramer-Rao lower bound.