1. *(from 3.97)* Let Y denote a random variable that has Poisson distribution with mean $\lambda = 2$. Find
   (a) $P(Y = 4)$
   (b) $P(Y \geq 4)$
   (c) $P(Y < 4)$
   (d) $P(Y \geq 4|Y \geq 2)$

2. **(3.103)** The number of knots in a particular type of wood has a Poisson distribution with an average of 1.5 knots in 10 cubic feet of wood. Find the probability that a 10-cubic-foot block of wood has at most one knot.

3. **(3.98, 99, 100)** Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that
   (a) no more than three customers arrive?
   (b) at least two customers arrive?
   (c) exactly five customers arrive?

   If it takes approximately 10 minutes to serve each customer,

   (d) find the mean and variance of the total time servers spend serving customers arriving during a 1-hour period. Assume there are enough servers so no customer must wait. *Hint:* Let X be the number of customers that arrive in the 1-hour period, and let $Y = 10X$ be the number of minutes servers spend serving these customers.
   (e) Is it likely that the total service time will exceed 2.5 hours (150 minutes). *Hint:* Use the empirical rule (on p. 10).

   Find the probability that *exactly* two customers arrive in the 2-hour period of time (leaving them as a mathematical expression is preferred).

   (f) between 2pm and 4pm (one continuous 2-hour period).
   (g) between 1pm and 2pm or between 3pm and 4pm (two separate 1-hour periods that total two hours).
   (h) How are these two probabilities related? Does this suggest a general rule to you?

4. **(3.106, 107)** Consider a binomial experiment for $n = 20$, $p = 0.05$. Use Table I to calculate the binomial probabilities for $Y = 0, 1, 2, 3, 4$. Calculate the same probabilities using the Poisson approximation with $\lambda = np$.

   A salesperson has found that the probability of a sale on a single contact is approximately 0.03. If the salesperson contacts 100 prospects, what is the approximate probability of making at least one sale? Calculate both exactly and using the Poisson approximation.

5. **(3.110)** Let Y have a Poisson distribution with mean λ. Find $E[Y(Y-1)]$ and use this to show that $V(Y) = \lambda$. *Hints:* Follow the technique used to find the mean on p. 128–129. Also, $E[Y(Y-1]) = EY^2 - EY$, which can be substituted carefully into the variance equation $V(Y) = EY^2 - (EY)^2$.

6. What are the two main applications of the moment-generating function?

7. **(3.115, 116, 119)** If Y has a binomial distribution with n trials and probability of success p

   (a) show that the moment-generating function for Y is
   $$m(t) = \left(pe^t + q\right)^n, \quad \text{where } q = 1 - p.$$
   Hint: Use the binomial expansion $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$.

   (b) Differentiate this moment-generating function to find EY and EY^2. Then find $V(Y)$.

   (c) Use the uniqueness of moment-generating functions to give the distribution of a random variable with moment-generating function $m(t) = (.6e^t + .4)^3$.

8. **(3.123)** Let $m(t) = (1/6)e^t + (2/6)e^{2t} + (3/6)e^{3t}$. Find the following:

   (a) $E(Y)$.

   (b) $V(Y)$.

   (c) the distribution of Y.

9. **(3.131)** Let Y be a random variable with mean 11 and variance 9. Using Tchebysheff's theorem, find

   (a) a lower bound for $P(6 < Y < 16)$,

   (b) the value of C such that $P(|Y - 11| \geq C) \leq 0.09$.

10. **(3.139)** The WSJ notes that hang gliding can be dangerous. Of the estimated 36,000 hang-gliding enthusiasts in the US, 29 were killed last year in hang-gliding accidents, and many more were injured. Suppose a randomly selected hang-gliding pilot has a probability 0.0006 of being killed in any one year. If there are 40,000 pilots next year,

    (a) what is the expected number of fatalities?

    (b) what is the standard deviation of of the number of fatalities?

    (c) using Tchebysheff's rule, is it likely that the number of fatalities would exceed 40?

*Problems noted with **(1.1)** or (from 1.1) are taken directly from or based on problems from our text, respectively.*