1 Let A be the event that the well has impurity A, and B be the event that the well has impurity B. Then we are told that $P(A) = 0.4$, $P(B) = 0.5$, and $P(\overline{A \cup B}) = 0.2$. Thus by the complement rule, $P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - 0.2 = 0.8$, and derived from the additive rule, the probability of getting both is $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5+0.4-0.8 = 0.1$. Finally, the probability of getting exactly one is the probability of getting at least one minus the probability of getting both: $P(A \cup B) - P(A \cap B) = 0.8 - 0.1 = 0.7$. So the probability distribution is

y	0	1	2
$p(y)$	0.2	0.7	0.1

and the expected value is $E(Y) = 0(0.2) + 1(0.7) + 2(0.1) = 0.9$.

2 There are $\binom{5}{2} = 10$ total possibilities with equal probability, so I'll just write them out:

	(1,2)	(1,3)	(1,4)	(1,5)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)
x	2	3	4	5	3	4	5	4	5	5
y	3	4	5	6	5	6	7	7	8	9

From these I can easily write out the probability distributions by counting the number of times each number occurs, they each have probability 0.1 as they have equal probability:

x	2	3	4	5
$p(x)$	0.1	0.2	0.3	0.4

and

y	3	4	5	6	7	8	9
$p(y)$	0.1	0.1	0.2	0.2	0.2	0.1	0.1

Then $E(X) = 2(0.1) + 3(0.2) + 4(0.3) + 5(0.4) = 4$
and $E(Y) = 3(0.1)+4(0.1)+5(0.2)+6(0.2)+7(0.2)+8(0.1)+9(0.1) = 24(0.1)+18(0.2) = 6$.
And for the variance, $E(X^2) = 2^2(0.1) + 3^2(0.2) + 4^2(0.3) + 5^2(0.4) = 17$
and $E(Y^2) = (3^2 + 4^2 + 8^2 + 9^2)(0.1) + (5^2 + 6^2 + 7^2)(0.2) = 17 + 22 = 39$.
So $V(X) = 17 - 4^2 = 1$ and $V(Y) = 39 - 6^2 = 3$.

3

$$\begin{aligned} V(aX + b) &= E[(aX + b - E(aX + b))^2] = E[(aX + b - aEX - b)^2] \\ &= E[a^2(X - EX)^2] = a^2E[(X - EX)^2] = a^2V(X) \end{aligned}$$

$$\begin{aligned} V(aX + b) &= E[(aX + b)^2] - [E(aX + b)]^2 = E(a^2X^2 + 2abX + b^2) - [aE(X) + b]^2 \\ &= a^2E(X^2) + 2abE(X) + b^2 - a^2[E(X)]^2 - 2abE(X) - b^2 \\ &= a^2E(X^2) - a^2[E(X)]^2 = a^2[E(X^2) - (E(X))^2] = a^2V(X) \end{aligned}$$

4 a. $E(X + Y) = EX + EY = 5 + 7 = 12$
b. $E(4X + Y) = 4EX + EY = 4(5) + 7 = 27$
c. $V(5X + 3) = 5^2V(X) = 25(3) = 75$
d. $V[(Y - 2)/4] = V(Y - 2)/4^2 = V(Y)/16 = 4/16 = 1/4$
or $= V(Y/4 - 1/2) = V(Y)/4^2 = 4/16 = 1/4$

5 $U \sim \text{NegBin}(0.3, 2)$, $V \sim \text{Bin}(0.1, 30)$, $W \sim \text{NegBin}(0.6, 5)$, $X \sim \text{Geo}(0.01)$, $Y \sim \text{Bin}(0.99, 10)$.

Z is not any of these because I'm sampling without replacement, so the probability of choosing a sharp pencil is not the same for each draw, as is required for each of these distributions. For example, on the first draw, the probability of choosing a sharp pencil is 2/10. But if I draw a dull one on the first draw, the probability of choosing a sharp pencil on the next draw is 2/9. If we've reached section 3.7, you'll recognize this as a hypergeometric distribution with $N = 10$ and $r = 2$.

6 Let X be the number of people that recover. Then $X \sim \text{Bin}(0.8, 20)$.

a. $P(X = 14) = P(X \leq 14) - P(X \leq 13) = .196 - .087 = .109$
b. $P(X \geq 10) = 1 - P(X < 10) = 1 - P(X \leq 9) = 1 - .001 = .999$
c. $P(10 \leq X \leq 18) = P(X \leq 18) - P(X \leq 9) = .931 - .001 = .930$
d. $P(X \leq 16) = .589$

7 This is the same as if he answers five or less incorrectly, which happens with probability 0.8. We turn it around this way so we can use Table 1 (p. 784), which says that this probability is 0.000 to three decimal places.

8 The number of successful operation X is $\text{Bin}(0.1, 10)$, so $E(X) = 10(0.1) = 1$ and $V(X) = 10(0.1)(0.9) = 0.9$. Let C_i be the cost for the ith expedition. Then the expected cost for the ith exploration, $E(C_i)$, is $30,000(0.1) + 15,000(0.9) = 16,500$. So the expected cost for ten explorations plus the preparation costs is $E(20,000 + \sum_{i=1}^{10} C_i) = 20,000 + \sum_{i=1}^{10} E(C_i) = 20,000 + 10E(C_1) = 20,000 + 10 \times 16,500 = 185,000$.

9 This is a $\text{Geo}(0.3)$ random variable, so $p(5) = 0.7^4 0.3 = 0.07203$, and the expected value is $1/0.3 = 3\frac{1}{3}$.

10

a. To be found on the second trial, the first is defective (0.1) and the second is not (0.9) so the probability is $0.1 \times 0.9 = 0.09$. Or, the number of trials to see the first nondefective engine is $\text{Geo}(0.9)$, and with this distribution, $p(2) = (0.1)^{2-1}(0.9) = 0.09$.
b. The number of trials needed to get three nondefective engines is $\text{NegBin}(0.9, 3)$, so the probability that it's on the fifth trial is $p(5) = \binom{4}{2} 0.9^3 0.1^2 = 0.04374$. The probability that it's on or before the fifth trial is the probability that it occurs on either the third, fourth, or fifth trial, which is
$\binom{2}{2} 0.9^3 + \binom{3}{2} 0.9^3 0.1^1 + \binom{4}{2} 0.9^3 0.1^2 = 0.729 + 0.2187 + 0.04374 = 0.99144$.

c. The number of trials on which the first nondefective engine is found is Geo(0.9), which has mean $1/0.9 = 10/9$ and variance $(1-0.9)/0.9^2 \approx 0.1235$.
The number of trials on which the third nondefective engine is found is NegBin$(0.9, 3)$, which has mean $3/0.9 = 3\frac{1}{3}$ and variance $(3(1-0.9))/0.9^2 \approx 0.370$.

d. Each engine is independent, so this is equivalent to the probability that at least two engines must be tested before the first nondefective is found. This is the complement of the event that only one engine must be tested, which has probability 0.9, so the desired probability is $1 - 0.9 = 0.1$.
We could also compute the conditional probability directly, where $X \sim \text{Geo}(0.9)$

$$P(X \geq 4 | X > 2) = \frac{P(X \geq 4)}{P(X > 2)} = \frac{1 - P(X \leq 3)}{1 - P(X \leq 2)} = \frac{1 - (P(X=1) + P(X=2) + P(X=3))}{1 - (P(X=1) + P(X=2))}$$
$$= \frac{1 - (0.9 + 0.1(0.9) + 0.1^2 0.9)}{1 - (0.9 + 0.1(0.9))} = \frac{.001}{.01} = 0.1$$

11 OK, I meant to give you this geometric series result (found in the Appendix), but evidently forgot. Here is the one that's useful to use here: $\sum_{k=0}^{n} x^k = \frac{1-x^{k+1}}{1-x}$ for $x \leq 1$. Using this,

$$P(Y > a) = 1 - P(Y \leq a) = 1 - \sum_{i=1}^{a} (1-p)^{i-1} p = 1 - p \sum_{j=0}^{j-1} (1-p)^j = 1 - p\frac{1-(1-p)^a}{1-(1-p)} = (1-p)^a.$$

Or this can be done quite straightforwardly using induction. For $a = 1$, $P(Y > 1) = 1 - P(Y = 1) = (1-p)^1$, which is the desired result. Then assuming $P(Y > a) = (1-p)^a$ is true for values from 1 to a, we find that $P(Y > a+1) = P(Y > a) - P(Y = a+1) = (1-p)^a - (1-p)^a p = (1-p)^a(1-p) = (1-p)^{a+1}$, thus proving the desired equality.

Using this result,

$$P(Y > a+b | Y > a) = \frac{P(Y > a+b \cap Y > a)}{P(Y > a)} = \frac{P(Y > a+b)}{P(Y > a} = \frac{(1-p)^{a+b}}{(1-p)^a} = (1-p)^b$$

It's called memoryless because what happened in the first a trials doesn't affect what happens in the next b trials.