1. *(from 3.1)* When the health department tested private wells in a county for two impurities commonly found in drinking water, it found that 20% of the wells had neither impurity, 40% had impurity A, and 50% had impurity B. (Obviously, some had both.) If a well is randomly chosen from those in the county, find the probability distribution for Y, the number of impurities found in the well (that is, no impurities, one impurity (either A or B), or two impurities (both A and B). Hint: use the probability rules to transform the probabilities you need into probabilities you are given. What is the expected number of impurities you find in this randomly selected well?

2. *(from 3.6)* Five balls, numbered 1, 2, 3, 4, and 5, are placed in an urn. Two balls are randomly selected from the five, and their numbers noted. Find the probability distribution, the expected value, and the variance for

   (a) the *largest* of the two sample numbers, X.
   (b) the *sum* of the two sample numbers, Y.

3. *(from 3.23)* Use the rules of expectation to prove the variance rule $V(aX + b) = a^2V(X)$.

4. Suppose $E(X) = 5$, $V(X) = 3$, and $E(Y) = 7$, $V(Y) = 4$. Find:

   (a) $E(X + Y)$
   (b) $E(4X + Y)$
   (c) $V(5X + 3)$
   (d) $V[(Y - 2)/4]$

5. Determine if the following random variables follow a Binomial, Geometric, or Negative Binomial distribution, and determine their parameters (n and p for Binomial, p for Geometric, and p and r for Negative Binomial):

   (a) A thumbtack will land with the point directly up 30% of the time. It is repeatedly flipped until it lands point up twice. Let U be the number of times to thumbtack is flipped.
   (b) A specific textbook has a 10% chance of falling apart after a semester of use. If a class of 30 all has this textbook, let V be the number of textbooks that have fallen apart by the end of the semester.
   (c) An archer hits her target with probability 0.6. Let W be the number of arrows she shoots in order to hit the target 5 times.
   (d) A machine that makes pencils makes *correctly made* pencils with probability 0.99. The boss asks to see what a defective pencil looks like, and waits until a *defective* pencil is made. Let X be the number of pencils that the boss watches made.
   (e) (continued from the last one. . .) If 10 pencils come in a box, let Y be the number of *correctly made* pencils in a given box.
   (f) I have 10 pencils in my drawer, but only two are sharpened. I randomly choose pencils from my drawer one at a time until I find a sharp one. I don't bother to return the unsharpened pencils to my drawer before choosing again. Let Z be the number of pencils I choose from my drawer. *Hint:* This is a trick question; it's not any of these three. Explain why not.

6. **(3.28)**The probability that a patient recovers from a stomach disease is 0.8. Suppose 20 people are known to have contracted the disease. What is the probability that . . .

   (a) exactly 14 recover?
   (b) at least 10 recover?
   (c) at least 10 but not more than 18 recover?
   (d) at most 16 recover?

7. **(3.29)**A multiple-choice exam has 15 questions, each with five possible answers, only one of which is correct. Suppose a student answers each question with an independent random guess. What's the probability that he answers at least ten questions correctly?

8. **(3.40, 3.41)**An oil exploration firm is formed with enough capital to finance ten explorations. The probability of an exploration being successful is 0.1. Assume the explorations are independent.

   (a) Find the mean and variance of the number of successful operations.
   (b) Suppose the team has a fixed cost of \$20,000 for preparing the equipment before doing any explorations, and then has a cost of \$30,000 for each successful operation and \$15,000 for each unsuccessful operation. Find the expected total cost to the firm for its ten explorations (including the preparation costs).

9. **(3.51,3.52)** Suppose that 30% of the applicants for a certain industrial job possess advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool.

   (a) Find the probability that the first applicant with advanced training in computer programming is found on the fifth interview.
   (b) What is the expected number of applicants who need to be interviewed in order to find the first one with advanced training?

10. **(3.74–3.77)** Ten percent of the engines manufactured on an assembly line are defective. The engines are randomly selected one at a time and tested.

    (a) What is the probability that the first nondefective engine is found on the second trial?
    (b) What is the probability that the third nondefective engine is found on the fifth trial? on or before the fifth trial?
    (c) Find the mean and variance of the number of trials on which the first nondefective engine is found, and on which the third nondefective engine is found.
    (d) Given that the first two engines tested were defective, what is the probability that at least two more engines must be tested before the first nondefective is found?

11. **(3.55)** Let Y denote a geometric random variable with probability of success p.

    (a) Show that for a positive integer a, $P(Y > a) = (1-p)^a$.
    (b) Show that for positive integers a and b,

    $$P(Y > a + b | Y > a) = (1-p)^b = P(Y > b).$$

    This result implies that, for example, $P(Y > 7|Y > 2) = P(Y > 5)$. Why do you think that this result is called the *memoryless* property of the geometric distribution?

*Problems noted with **(1.1)** or (from 1.1) are taken directly from or based on problems from our text, respectively.*