Solution of HW 3

(10) Problem 1.

(a) $$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$$

(b) $$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{0.1}{0.5} = \frac{1}{5}$$

(c) $$P(A|A\cup B) = \frac{P(A\cap(A\cup B))}{P(A\cup B)} = \frac{P(A)}{P(A)+P(B)-P(A\cap B)} = \frac{0.5}{0.7} = \frac{5}{7}$$

(d) $$P(A|A\cap B) = \frac{P(A\cap(A\cap B))}{P(A\cap B)} = \frac{P(A\cap B)}{P(A\cap B)} = 1$$

(e) $$P(A\cap B|A\cup B) = \frac{P((A\cap B)\cap(A\cup B))}{P(A\cup B)} = \frac{P(A\cap B)}{P(A\cup B)} = \frac{0.1}{0.7} = \frac{1}{7}$$

(10) Problem 2.

(a) $$P(A\cap B) = P(A)+P(B)-P(A\cup B) = 0.2+0.3-0.4 = 0.1$$

(b) $$P(\bar{A}\cup\bar{B}) = P(\overline{A\cap B}) = 1-P(A\cap B) = 1-0.1 = 0.9$$

(c) $$P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1-P(A\cup B) = 1-0.4 = 0.6$$

(d) $$P(\bar{A}|B) = P(B\cap\bar{A})/P(B) = \frac{P(B)-P(B\cap A)}{P(B)} = \frac{0.3-0.1}{0.3} = \frac{2}{3}$$

(10) Problem 3.

(a) $P(A\cup B) = P(A)+P(B) - P(A\cap B)$

$\overset{\text{indep}}{=} P(A)+P(B) - P(A)\cdot P(B)$

$= 0.5+0.2-0.5\times 0.2$

$= 0.6$

(b) $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1-P(A\cup B) = 1-0.6 = 0.4$

(c) $P(\bar{A}\cup\bar{B}) = P(\overline{A\cap B}) = 1-0.1 = 0.9$

Problem 4.

(i) To show (a) $\Rightarrow$ (b)

In fact, if (a) holds, then

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \overset{(a)}{=} \frac{P(A)\cdot P(B)}{P(B)} = P(A)$$

(ii) To show (b) $\Rightarrow$ (c)

In fact, if (b) holds, then

$$P(B|A) = \frac{P(A|B)\cdot P(B)}{P(A)} \underset{(b)}{=} \frac{P(A)\cdot P(B)}{P(A)} = P(B)$$

(iii) To show (c) $\Rightarrow$ (a)

In fact, if (c) holds, then

$$P(A\cap B) = P(B|A)P(A) \underset{(c)}{=} P(B)\cdot P(A)$$

All in all, (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (a).

Problem 5.

since $P(A|B) > P(A)$

$\Rightarrow \dfrac{P(A \cap B)}{P(B)} > P(A)$

$\Rightarrow \dfrac{P(A \cap B)}{P(A)} > P(B)$

$\Rightarrow P(B|A) > P(B)$.

Problem 6:

P (at least one of the three events occuring)

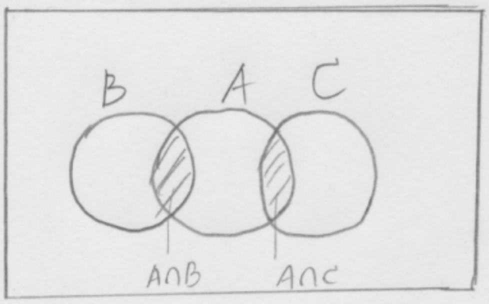
$= P(A \cup B \cup C)$

$= P(A)+P(B)+P(C) - P(A\cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= P(A)+P(B)+P(C) - P(A \cap B) - 0 - P(A \cap C) + 0$

$= P(A)+P(B)+P(C) - P(A \cap B) - P(A \cap B)$

$= P(A)+P(B)+P(C) - 2P(A \cap B)$



Problem7:

Define $S = \{ \text{satisfied job} \}$

$A = \{ \text{job did by plumber } A \}$

then we know

$P(\bar{S}) = 0.1$

$P(A) = 0.4$

$P(A \mid \bar{S}) = 0.5$

(a) $P(\bar{S} \mid A) = \dfrac{P(\bar{S} \cap A)}{P(A)} = \dfrac{P(A \mid \bar{S})\, P(\bar{S})}{P(A)} = \dfrac{0.5 \times 0.1}{0.4} = 0.125$

(b) $P(S \mid A) = 1 - P(\bar{S} \mid A) = 1 - 0.125 = 0.875$

Problem 8:

Denote: $A = \{ \text{will contract disease I} \}$

$B = \{ \text{will contract disease II} \}$

we have $P(A) = 0.1$, $P(B) = 0.15$, $P(A \cap B) = 0.03$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.1 + 0.15 - 0.03$

$= 0.22$

(b) $P(A \cap B \mid A \cup B) = \dfrac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A \cap B)}{P(A \cup B)} = \dfrac{0.03}{0.22} = 0.136$

Problem 9 :

Denote $M = \{$ Person sees a magazin ad $\}$

$T = \{$ Person sees a TV ad $\}$

$B = \{$ Person buys the product $\}$

Then we know.

$P(M) = \frac{1}{50} = 0.02$, $P(T) = \frac{1}{5} = 0.2$

$P(M \cap T) = \frac{1}{100} = 0.01$

$P(B \mid M \cup T) = \frac{1}{3} = 0.33$

$P(B \mid \overline{M \cup T}) = \frac{1}{10} = 0.1$

$\Rightarrow$ $P(M \cup T) = P(M) + P(T) - P(M \cap T) = 0.02 + 0.2 - 0.01 = 0.21$

$P(\overline{M \cup T}) = 1 - P(M \cup T) = 1 - 0.21 = 0.79$

$\Rightarrow$ $P(B) = P(B \mid M \cup T) \cdot P(M \cup T) + P(B \mid \overline{M \cup T}) P(\overline{M \cup T})$

$= \frac{1}{3} \times 0.21 + 0.1 \times 0.79$

$= 0.149$

(5) Problem 10 :

Define : $A = \{$ person indeed has disease $\}$

$B = \{$ The diagnostic test indicates one has disease $\}$

$$P(A|B) = \frac{P(A)\cdot P(B|A)}{P(B)}$$

$$= \frac{P(A)\cdot P(B|A)}{P(A)\cdot P(B|A) + P(\bar{A})\cdot P(B|\bar{A})}$$

$$= \frac{0.01 \times 0.9}{0.01\times 0.9 + 0.99\times 0.1} = \frac{1}{12}$$

(10) Problem 11.

Denote $A = \{$an aircraft is detected by $A\}$

$B = \{$an aircraft is detected by radar $B\}$

$C = \{$an aircraft is detected by radar $C\}$

$\Rightarrow P(A) = P(B) = P(C) = 0.98$

(a) $P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C}) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$$\overset{\text{indpt}}{=} 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$$

$$= 1 - (1 - 0.98)^3$$

$$= 0.999992$$

(b) $P(A \cap B \cap C) \underset{\text{indpt}}{=} P(A)\cdot P(B)\, P(C) = 0.98^3 = 0.9412$

(c) $P(\text{at least one is not dected}) = 1 - P(\text{all are detected})$

$$= 1 - P(\text{first craft is deteted})^3$$

$$= 1 - P(A \cup B \cup C)^3$$

$$= 1 - 0.999992^3 = 0.000024$$

(5) Problem 12.

Denote $B_i = \{ \text{Bowl } i \text{ is selected} \}$, $i = 1, 2, 3, 4, 5$

$W = \#$ of white balls in selected balls

Then $P(B_i) = \frac{1}{5}$, $i = 1, \cdots, 5$

(a) $P(W=2)$

$= \sum_{i=1}^{5} P(W=2 \mid B_i) P(B_i)$

$= 0 \times \frac{1}{5} + \frac{1}{\binom{5}{2}} \times \frac{1}{5} + \frac{\binom{3}{2}}{\binom{5}{2}} \times \frac{1}{5} + \frac{\binom{4}{2}}{\binom{5}{2}} \times \frac{1}{5} + 1 \times \frac{1}{5}$

$= 0.4$

(b) $P(B_3 \mid W=2)$

$= \dfrac{P(B_3) \cdot P(W=2 \mid B_3)}{P(W=2)}$

$= \dfrac{\frac{1}{5} \times \frac{\binom{3}{2}}{\binom{5}{2}}}{0.4}$

$= 0.15$