Solution of HW2

(5)

Problem 1:

(a) not allowed, because $P(B)$ is negative

(b) not allowed, because $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 1.1 > 1$.

(c) allowed. eg. $P(A) = 0.5$ $P(B) = 0.6$, $A \subseteq B \Rightarrow P(A\cap B) = 0.5$

Problem 2:

(a) $S = \{A, B, AB, O\}$

(b) $P(A) = 0.41$, $P(B) = 0.10$

$P(AB) = 0.04$, $P(O) = 0.45$

(c)
$$\begin{aligned} P(A \cup AB) &= P(A) + P(AB) - P(A\cap AB) \\ &= P(A) + P(AB) - P(\emptyset) \\ &= 0.41 + 0.04 - 0 \\ &= 0.45 \end{aligned}$$

(5)

Problem 3:

(a) not disjoint, because a person can need glassed and use them for reading

(b) $P(N) = 0.44 + 0.14 = 0.58$

(c) $P(N\cap R) = 0.44$

(d) $P(N\cap \bar{R}) = 0.14$

(e) $P(\bar{N}\cap \bar{R}) = 0.4$

Problem 4:

B = bushing defected $\quad$ S = shaft defected

we know

$P(B \setminus S) = 0.06$, $\quad P(S \setminus B) = 0.08$

$P(B \cap S) = 0.02$

(a) $P(B) = P(B \setminus S) + P(B \cap S) = 0.02 + 0.06 = 0.08$

(b) $P(B \cup S) = P(B \setminus S) + P(B \cap S) + P(S \setminus B)$

$= 0.06 + 0.08 + 0.02$

$= 0.16$

(c) $P((B \setminus S) \cup (S \setminus B)) = P(B \setminus S) + P(S \setminus B) = 0.06 + 0.08 = 0.14$

(d) $P(\overline{B \cup S}) = 1 - P(B \cup S) = 1 - 0.16 = 0.84$

(5)

Problem 5: (2.22)

let A, B, C denote the three wines,

(a) One sample point is CBA, where C is 1st place or best, B is second best, A is worst.

(b) $S = \{ABC, ACB, BAC, BCA, CAB, CBA\}$

(c). WLOG, assume A is the best wine, the possibilities of A being no worse than second best = $\{ABC, ACB, BAC, CAB\}$

$\therefore$ probability $= \frac{4}{6} = \frac{2}{3}$

Problem 6: (2.30)

$$\overset{\text{App}}{\binom{4}{1}} \times \overset{\text{Sal}}{\binom{3}{1}} \times \overset{\text{Ent}}{\binom{4}{1}} \times \overset{\text{Des}}{\binom{5}{1}} = 4 \times 3 \times 4 \times 5 = 240$$

(5)

Problem 7: (2.34)

10 choose 3 with no replacement, the order does matter because it is different positions.

$$P_3^{10} = \frac{10!}{7!} = 720.$$

Problem 8: (2.35)

(a) Partion 9 taxis into three groups, group sizes are 3, 5, 1.

$$\binom{9}{3,5,1} = \frac{9!}{3!5!1!} = 504$$

Problem 9: (2.36)

(a) Given the information in previous probbem, the probability of exactly one taxi in need of repair going to airport C is 1/9. Since there are total of 9 taxis altogether and one needs repair, only one taxi can goto Airport C at one time, the chance of this one repair-needing taxi going to C is 1/9

(b). the ways of allocating 6 taxis (no need of repair) is

$$\binom{6}{2\ 4\ 0} = \frac{6!}{2!4!} = 15$$

the way of allocating 3 repair-needing taxi is $P_3^3 = 6$

$$\text{prob} = \frac{15 \times 6}{504} = \frac{5}{28}$$

Problem 10: (2.39)

(a). $\binom{130}{2} = \frac{130!}{2!\,128!} = 8385$

(b). Use the mn rule; since 26 letters available, we have

$\underbrace{(26*26)}_{\text{two-letter code}} + \underbrace{(26*26*26)}_{\text{three-letter code}} = 18252$ major codes

(c) $\binom{130}{1} + \binom{130}{2} = 8515.$

(d). Yes, because $18252 > 8515$.

Problem 11: (2.44)

(5) total # of sample points $= \binom{8}{4} = \frac{8!}{4!\,4!} = 70$

total # of points in event $= \underbrace{\binom{3}{2}}_{\text{undergraduate}} * \underbrace{\binom{5}{2}}_{\text{graduate}} = \frac{3!}{2!\,1!} \cdot \frac{5!}{2!\,3!} = 30$

Prob $= 30/70 = 3/7$

Problem 12: (2.50)

(2.5) total # of sample points $= 6^6 = 46656$

total # of points in event $= 6! = 720$

Prob $= 720/46656 = 5/324$

Problem 13: (2.51)

(2.5) total of sample points $= \frac{6^5}{2!} = 3888$

total of event $= 5! = 120$

$$\text{Prob} = \frac{120}{3888} = \frac{5}{162}$$