1. (a) Setting $P(0 < X < 2) = 1$ and solving for c, $1 = \int_0^2 cx^2 dx = \left.\frac{cx^3}{3}\right|_0^2 = c\left(\frac{8}{3}\right)$, so $c = \frac{3}{8}$.

   (b) For $0 < x < 2$, $F(x) = \int_0^x \frac{3}{8}x^2 dx = \left.\frac{x^3}{8}\right|_0^x = \frac{x^3}{8}$, so

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0, \\ \frac{x^3}{8} & \text{for } 0 < x < 2, \\ 1 & \text{for } x \geq 2. \end{cases}$$

   (c) $P(X > 1) - 1 - F(1) = 1 - \frac{1^3}{8} = \frac{7}{8}$.

   (d) $E\left(\frac{1}{x}\right) = \int_0^2 \frac{1}{x}\left(\frac{3}{8}x^2\right) dx = \frac{3}{8}\int_0^2 x dx = \left.\frac{3}{8}\frac{x^2}{2}\right|_0^2 = \frac{3}{4}$.

2. (a) $(120 - 100)/20 = 1$; from the table this gives 15.87%.

   (b) From the table, 25% of the scores are above 0.67, so by symmetry, 75% of the scores are above -0.67. Then the cutoff is $100 + (-0.67) \times 20 = 86.6$.

   (c) Letting X be the score, $P(X > 100|\text{in school B}) = P(X > 100)/P(\text{in school B}) = 0.5/0.75 = 2/3$.

3. (a) $m_X(t) = (1-t)^{-1}$ corresponds to a Exponential with $\beta = 1$.

   (b) $a = 0$ and $b = 2$, so $m_y(t) = e^{0t}m_X(2t) = (1-2t)^{-1}$.

   (c) This is an Exponential with $\beta = 2$.

4. (a) $P(Y = 1) = 0.2 + 0.1 = 0.3$, so

x	0	1
$p(x\|Y=1)$	$\frac{0.2}{0.3} = \frac{2}{3}$	$\frac{0.1}{0.3} = \frac{1}{3}$

   (b) $E(X|Y = 1) = 0\left(\frac{2}{3}\right) + 1\left(\frac{1}{3}\right) = \frac{1}{3}$.

   (c) X and Y are not independent: $p_X(0)p_Y(0) = (0.5)(0.7) = 0.35 \neq 0.3 = p(0,0)$. This would have to be true for all values of x and y; since we found one where it is not, they are not independent.

5. (a) Yes, X and Y are independent. The support is rectangular, and the density can be factored into a part that depends only on x and a part that depends only on y; in this it's a constant, so, for example, if $g(x) = 1$ and $h(y) = \frac{1}{2}$, $f(x,y) = g(x)h(y)$.

   (b) $P\left(X < 0, Y < \frac{1}{2}\right)$ is the volume above that area. The area is $1 \times \frac{1}{2} = \frac{1}{2}$, and the height is $\frac{1}{2}$, so the volume is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

   (c) To find $E(Y|X)$, we consider that we're on a line for any given x value; if that's true then y can be any value between 0 and 1, and since the density is uniform, the expected value (the average) is $\frac{1}{2}$.

6. (a) X and Y are not independent, as the support is not rectangular.

   (b) $f_Y(y) = \int_x f(x,y)dx = \int_0^{3-y} \frac{1}{2(3-y)}dy$. Evaluating this (which I didn't ask you to do) gives

$$f_Y(y) = \left.\frac{x}{2(3-y)}\right|_{x=0}^{x=3-y} = \frac{3-y}{2(3-y)} = \frac{1}{2}.$$

   (c) The support for the marginal of y is the range of possible values for y; it has nothing to do with x, so the support is $0 < y < 2$.

   (d) $f(x|y) = \frac{f(x,y)}{f_Y(y)} = \left(\frac{1}{2(3-y}\right) / \left(\frac{1}{2}\right) = \frac{1}{3-y}$, for $0 < x < 3-y$. The possible values for y (nice, but not required) are $0 < y < 2$.