1a D, R, and I partition the set of Minnesotans, so by the law of total probability,

$$P(A) = P(A|D)P(D) + P(A|R)P(R) + P(A|I)P(I).$$

We know that $P(D) + P(R) + P(I) = 1$. Let $p = P(D)$. Then

$$P(R) = 1 - P(I) - P(D) = 1 - 0.4 - p = 0.6 - p,$$

so substituting this in, as well as all that we given, we have

$$0.3 = 0.05p + 0.7(0.6 - p) + 0.25(0.4).$$

Solving for p, we find that $p = 0.338$. So $P(D) = p = 0.338$ and $P(R) = 0.6 - p = 0.262$.

1b By Bayes' Rule,

$$P(D|A) = \frac{P(A|D)P(D)}{P(A)} = \frac{0.05(0.338)}{0.3} = 0.056$$

2a No. If they were mutually exclusive, $P(A \cup B) = P(A) + P(B) = 1.1$, which is impossible.

2b By independence, $P(A \cap B) = P(A)P(B) = 0.6 \cdot 0.5 = 0.3$.

2c $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.6 - 0.3 = 0.8$.

2d Now $P(A \cap B) = P(B|A)P(A) = 0.3 \cdot 0.6 = 0.18$, so $P(A \cup B) = 0.5 + 0.6 - 0.18 = 0.92$.

3a $E(X) = \sum_x xp(x) = -1(0.4) + 0(0.4) + 2(0.2) = 0$

3b $E(X^2) = \sum_x x^2 p(x) = (-1)^2(0.4) + 0^2(0.4) + 2^2(0.2) = 0.4 + 0.8 = 1.2$, so $\text{Var}(X) = E(X^2) - (EX)^2 = 1.2 - 0^2 = 1.2$.

3c $E(X^3) = \sum_x x^3 p(x) = (-1)^3(0.4) + 0^3(0.4) + 2^3(0.2) = -0.4 + 1.6 = 1.2$

3d $E(e^{tX}) = \sum_x e^{tx} p(x) = 0.4e^{-t} + 0.4 + 0.2e^{2t}$

3e $m'(t) = -0.4e^{-t} + 2 \cdot 0.2e^{2t}$
$m''(t) = 0.4e^{-t} + 2^2 \cdot 0.2e^{2t}$
$m'''(t) = -0.4e^{-t} + 2^3 \cdot 0.2e^{2t}$
so $E(X^3) = m'''(0) = -0.4 + 8(0.2) = 1.2$.

4a $E(2X - 1) = 2EX - 1 = 2(3) - 1 = 5$

4b $\text{Var}(2X - 1) = 2^2 \, \text{Var}(X) = 2^2(9) = 36$

4c $\text{Var}(X) = E(X^2) - (EX)^2$, so $E(X^2) = \text{Var}(X) + (EX)^2 = 9 + 3^2 = 18$.
Then $E(3X - X^2) = 3EX - E(X^2) = 3(3) - 18 = -9$.

5a $\binom{10}{5}\binom{8}{4} = 252 \cdot 70 = 17640$

5b $\binom{2}{2}\binom{10}{5}\binom{6}{2} = 252 \cdot 15 = 3780$

5c This means that both brothers are not together on the committee, so there are $17640 - 3780 = 13860$ ways.

6 This is the complement of getting no threes, so the probability is $1 - (5/6)^6$.

7a $X \sim \text{NegBin}(100, 0.9)$

7b $X \sim \text{Bin}(10, 0.1)$

7c This is a Poisson random variable, with an average of one event every five feet. So in fifty feet of rope, there are an average of ten events, thus $X \sim \text{Poi}(10)$.

7d $X \sim \text{Geo}(0.6)$

8a $P(Y < 7) = P(Y \leq 6) = 0.610$

8b $P(Y > 4) = 1 - P(Y \leq 4) = 1 - 0.217 = 0.793$

8c $P(Y > 4|Y < 7) = \frac{P(4<Y<7)}{P(Y<7)} = \frac{P(Y<7)-P(Y\leq 4)}{P(Y<7)} = \frac{0.610-0.217}{0.610} = 0.644$