The exam is closed-book, but you may bring one sheet of letter-sized paper with notes on both sides. You will need a calculator.

1. In the recent *Minnesota Poll* conducted by the Star Tribune newspaper and reported on October 3, only 30% of Minnesotans approve of the way George W. Bush is handling his job as president. (The remaining 70% either disapprove, don't know, or refused to answer.) Broken down by party affiliation, 5% of Democrats, 70% of Republicans, and 25% of Independents approve. (Independents include everyone who is not either a Democrat or a Republican.)

   (a) Assuming 40% of Minnesotans are Independents (in 2004 it was 38%), show that 33.8% of Minnesotans are Democrats and 26.2% are Republicans.
   (b) You meet a Minnesotan who approves of Bush. Using the result in part a, what is the probability that this person is a Democrat? (Assume you met this person randomly.)

2. Consider two events: $A$ and $B$. We know that $P(A) = 0.6$ and $P(B) = 0.5$.

   (a) Are the events mutually exclusive? How can you tell?
   (b) Suppose that the events A and B are independent. What is the probability they occur simultaneously?
   (c) Again suppose that the events A and B are independent. What is the probability that at least one occurs?
   (d) Now suppose that $P(B|A) = 0.3$. What is the probability that at least one of the events occurs?

3. Let the random variable $X$ have the following distribution

| $x$ | $-1$ | $0$ | $2$ |
|---|---|---|---|
| $p(x)$ | 0.4 | 0.4 | 0.2 |

   (a) Find $E(X)$.
   (b) Find $\mathrm{Var}(X)$.
   (c) Find $E(X^3)$ directly.
   (d) Find the moment generating function for $X$, $E(e^{tX})$.
   (e) Use the mgf to find $E(X^3)$.

4. Let $E(X) = 3$ and $\mathrm{Var}(X) = 9$. Compute

   (a) $E(2X - 1)$
   (b) $\mathrm{Var}(2X - 1)$
   (c) $E(3X - X^2)$

5. A committee of 5 women and 4 men are to be selected from 10 women and 8 men.

   (a) How many total committees can be formed?
   (b) Suppose two of the men are brothers, and both insist on being on the committee. How many committees can be formed with both brothers included?
   (c) Suppose instead that the two brothers refuse to work together, so you cannot put them both on the committee. How many possible committees can be formed?

6. Suppose you independently roll a fair die 6 times. What is the probability that you get at least one "3" in the six outcomes?

7. What is the distribution of $X$ in each of the following situations? If the distribution has a name, please state it, and if it has parameters, please state those also. If the distribution has a shorthand notation, you may use that, for example, $X \sim \text{Bin}(5, 0.4)$. (In some situations, the distribution may not be exact. In that case, choose the distribution that best fits the situation.)

   (a) A machine produces parts one at at a time, and each part is judged to be either good or defective. The probability of a defective part is 0.1. The operator needs to produce 100 good parts to fill his quota for the day. Let $X$ be the total number of parts he makes in order to fill his quota.
   (b) Same machine and operator as above. The boss stops by to watch, and stays while 10 parts are made. Let $X$ be the number of bad parts the boss sees made.
   (c) Flaws occur in a rope at random, but on average are five feet apart. Let $X$ be the number of flaws in a fifty foot piece of rope.
   (d) Suppose 60% of all cars passing a certain point are speeding. A police officer measures the speed of each car passing that point. Let $X$ be the number of cars that the police officer measures in order to catch one speeding car.

8. Suppose $Y$ is a Binomial random variable with $n = 15$ and $p = 0.4$. Find

   (a) $P(Y < 7)$
   (b) $P(Y > 4)$
   (c) $P(Y > 4 | Y < 7)$