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Image Restoration

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Abstract

True images are usually degraded during image acquisition. Image restoration is for restoring true images from their observed but degraded versions; it is often used for preprocessing observed images so that subsequent image processing and analysis becomes more reliable. Among many different types of degradations, point degradations (or, noise) and spatial degradations (or, blurring) are most common in applications. This article introduces some fundamental image denoising and image deblurring methods.

Observed images generated by image acquisition devices are usually not exactly the same as the true images, but are instead degraded versions of their true images[10][19]. Degradations can occur in the entire process of image acquisition, and there are many different sources of degradation. For instance, in aerial reconnaissance, astronomy, and remote sensing, images are often degraded by atmospheric turbulence, aberrations of the optical system, or relative motion between the camera and the object. Image degradations can be classified into several categories, among which *point degradations* (or, noise) and *spatial degradations* (or, blurring) are most common in applications. Other types of degradations involve chromatic or temporal effects. For a detailed discussion about formation and description of various degradations, read [1].

Image restoration is a process to restore an original image f from its observed but degraded version Z. Since edges are important structures of the true image, they should be preserved during image restoration. In the literature, a commonly used model for describing the relationship between f and Z is

$$Z(x,y) = h \otimes f(x,y) + \varepsilon(x,y), \quad \text{for } (x,y) \in \Omega, \tag{1}$$

where h is a 2-D *point spread function (psf)* describing the spatial blur, $\varepsilon(x,y)$ is a pointwise noise at (x,y), Ω is the design space, and $h \otimes f$ denotes the convolution between h and f. In model (1), the spatial blur is assumed to be linear and location invariant, and the pointwise noise is additive, which may not be true in certain applications. See [23] for related discussion. Generally speaking, edge-preserving image restoration is challenging, due mainly to the facts that spatial blur and pointwise noise often mix up, it is difficult to remove them simultaneously, and the edge structure is hidden in the observed image intensities. Next, we describe certain fundamental image denoising and image deblurring methods, and provide some concluding remarks at the end.

Image Denoising

When there is no blurring in the observed image, $h \otimes f(x, y)$ is simply $f(x, y)$ and model (1) is a 2-dimensional nonparametric regression model. In such cases, our major goal is to estimate the true image f from its noisy version Z, or to do *image denoising*.

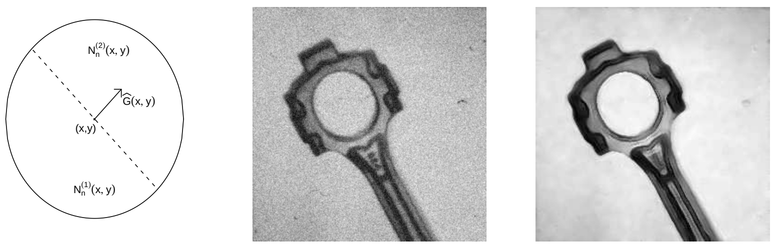
Image denoising is equivalent to estimating a jump surface from noisy data, because the true image can be regarded as a surface of the image intensity function f which has jumps at the outlines of image objects (or, at edges). To estimate f, conventional smoothing techniques would blur edges when removing noise. Recently, *jump regression analysis (JRA)*[19] is under rapid development, which provides smoothing methods that would preserve edges when estimating f. There are two types of such methods in the literature. By the first type, edges need to be detected before denoising. For instance, the three-stage procedure [17] works as follows. After edge detection, a principal component line is fitted through the center of detected edge pixels in a neighborhood of a given point, for approximating the underlying edge segment in the neighborhood. Then, observed image intensities on the same side of the principal component line, as the given point, are weighted averaged for estimating f at the given point. By the second type of methods, images can be restored properly without detecting edges explicitly (e.g., [5][8][15][18][22]). For instance, Gijbels et al. [8] define the edge-preserving estimator of f as follows. In a neighborhood $N_n(x, y)$ of a given point (x, y), we consider the following local linear kernel smoothing procedure:

$$\min_{a,b,c} \sum_{i=1}^{n} \{Z(x_i, y_j) - [a + b(x_i - x) + c(y_j - y)]\} K \left(\frac{x_i - x}{h_n}, \frac{y_j - y}{h_n} \right), \qquad (2)$$

where K is a circularly symmetric density kernel function with unit circular support, and h_n is a bandwidth. The solution to a of (2), denoted as $\widehat{a}_c(x, y)$, is the conventional local linear kernel (LLK) estimator of $f(x, y)$, and the solution to $(b, c)'$ is the LLK estimator, denoted as $\widehat{G}(x, y)$, of the gradient $G(x, y) = (f'_x(x, y), f'_y(x, y))'$. Then, $N_n(x, y)$ is divided into two halves $N_n^{(1)}(x, y)$ and $N_n^{(2)}(x, y)$ by a line perpendicular to $\widehat{G}(x, y)$, as demonstrated in Figure 1(a). Two one-sided surface estimators $\widehat{a}_1(x, y)$ and $\widehat{a}_2(x, y)$ are obtained, respectively, in the two halves of $N_n(x, y)$, by local linear kernel smoothing. The final surface estimator is defined by

$$\widehat{f}(x,y) = \begin{cases} \widehat{a}_c(x,y), & \text{if } \mathrm{dif}(x,y) \leq u \\ \widehat{a}_1(x,y), & \text{if } \mathrm{dif}(x,y) > u \text{ and } \mathrm{WRMS}_1(x,y) < \mathrm{WRMS}_2(x,y) \\ \widehat{a}_2(x,y), & \text{if } \mathrm{dif}(x,y) > u \text{ and } \mathrm{WRMS}_1(x,y) > \mathrm{WRMS}_2(x,y) \\ \frac{\widehat{a}_1(x,y)+\widehat{a}_2(x,y)}{2}, & \text{if } \mathrm{dif}(x,y) > u \text{ and } \mathrm{WRMS}_1(x,y) = \mathrm{WRMS}_2(x,y), \end{cases} \quad (3)$$

where $\mathrm{dif}(x,y) = \max\{\mathrm{WRMS}_c(x,y) - \mathrm{WRMS}_1(x,y), \mathrm{WRMS}_c(x,y) - \mathrm{WRMS}_2(x,y)\}$, $\mathrm{WRMS}_c(x,y)$, $\mathrm{WRMS}_1(x,y)$, and $\mathrm{WRMS}_2(x,y)$ denote the weighted residual mean squares of the corresponding fitted local planes, and u is a threshold parameter. Figures 1(b) and 1(c) show a noisy image and its denoised image by (3). For a more complete discussion about jump surface estimation and its application in image restoration, see [20].



(a) (b) (c)

Figure 1: (a) Neighborhood $N_n(x,y)$ of the point (x,y) is divided into two parts $N_n^{(1)}(x,y)$ and $N_n^{(2)}(x,y)$ by a line perpendicular to $\widehat{G}(x,y)$; (b) a noisy image; (c) denoised image.

In the computer science literature, image restoration by Markov Random Field (MRF) modeling is an active research area, which can remove noise and deblur images when the psf is known. Geman and Geman [7] provide a general framework for this approach as follows. First, the true image is assumed to be a MRF, and the observed image intensities are assumed to have a given conditional distribution conditional on the true image. Then, the true image is estimated by maximizing *a posteriori* (MAP). Generalizations and modifications are discussed by many authors. See, e.g., [2][9]. Other popular denoising methods include local median filtering [13], adaptive smoothing filtering [25], bilateral filtering [26], diffusion filtering [14], wavelet filtering [16], among some others.

Image Deblurring

The image deblurring problem, described by model (1), is generally ill-posed in the sense that (i) there might be many different sets of h and f corresponding to the same observed image Z, and (ii) the inverse problem to estimate f from Z often involves some numerical singularities (see related discussion below). Therefore, it is difficult to estimate both h and f properly from Z alone, without using any extra information about either f or h or both.

Early image deblurring methods assume that h is known. In such cases, f can be estimated based on the relationship that

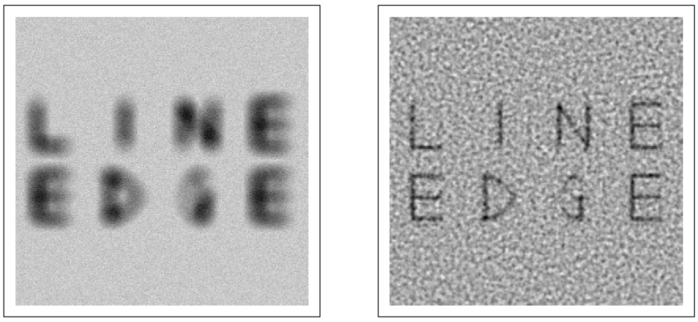
$$\mathcal{F}\{Z\}(u,v) = \mathcal{F}\{h\}(u,v)\mathcal{F}\{f\}(u,v) + \mathcal{F}\{\varepsilon\}(u,v), \text{ for } (u,v) \in R^2, \tag{4}$$

where $\mathcal{F}\{f\}$ denotes the Fourier transformation of f. In equation (4), if the noise term is ignored, then $\mathcal{F}\{f\}$ can be estimated easily by

$$\mathcal{F}\{f\}(u,v) = \frac{\mathcal{F}\{Z\}(u,v)}{\mathcal{F}\{h\}(u,v)}. \tag{5}$$

Then, an estimator of f can be obtained accordingly, using an inverse Fourier transformation. However, the noise effect would dominate this estimator because $\mathcal{F}\{h\}(u,v)$ usually converges to zero rapidly, as $u^2 + v^2$ tends to infinity, but $\mathcal{F}\{\varepsilon\}(u,v)$, which is part of $\mathcal{F}\{Z\}(u,v)$, converges to zero much slower. Consequently, the image estimator by (5) would be numerically unstable. In the literature, many proposals have been suggested to overcome this difficulty, including some non-iterative methods, such as the inverse filtering, Wiener filtering, and constrained least squares filtering procedures (cf., Chapter 5, [10]), and some iterative methods, such as the Lucy-Richardson procedure, Landweber procedure, Tikhonov-Miller procedure, MAP procedure, maximum entropy procedure, procedures based on EM algorithm, and so forth (e.g., [6]).

In many applications, however, it is difficult to specify psf h completely, based on our prior knowledge about the image acquisition device. Image deblurring when h is unknown is often referred to as the *blind image deblurring* problem. In the literature, a number of procedures have been proposed for solving this problem, which can be grouped roughly into three categories. One type of such procedures assumes that h can be described by a parametric model with one or more unknown parameters, and then the parameters together with the true image are estimated by some algorithms (e.g., [3][11]). The second type of procedures does not make restrictive assumptions on h, but they assume that the true image has one or more regions with certain known edge structures (e.g., [12][21]). For instance, the method by Qiu [21] assumes that the true image has one or more regions in which line edges are surrounded by uniform backgrounds. From these regions, the psf h can be estimated. Then, the entire image can be deblurred using the estimated h. Figure 2(a) shows a blurred and noisy image of the words "LINE EDGE." Figure 2(b) shows the deblurred image, using the estimated psf obtained from a small region surrounded the letter "I." The third type of blind image deblurring procedures try to avoid restrictive assumptions on both f and h. Instead, they put certain regularization measures on f and h to make the "ill-posed" deblurring problem solvable. For instance, the *total variation (TV)* method [4][24] formulates the



(a) (b)

Figure 2: (a) A blurred and noisy image; (b) deblurred image using the estimated psf obtained from a small region surrounded the letter “I”.

blind deblurring problem as

$$\min_{\widetilde{f},\widetilde{h}}\left\{ \quad \frac{1}{2}\int_{\Omega}\left[(\widetilde{h}\otimes\widetilde{f})(x,y)-Z(x,y)\right]^2\, dxdy + \right. \\ \left. \alpha_1\int_{\Omega}\left|\bigtriangledown\widetilde{f}(x,y)\right|\, dxdy+\alpha_2\int_{\Omega}\left|\bigtriangledown\widetilde{h}(x,y)\right|\, dxdy\right\}, \qquad (6)$$

where α_1 and α_2 are two positive parameters, and $|\bigtriangledown\widetilde{f}(x,y)|$ is the gradient magnitude of $f(x,y)$. Then, solutions of (6) to $\widetilde{f}$ and $\widetilde{h}$ are used as estimators of f and h, respectively. Clearly, in (6), the first term measures the goodness-of-fit of the estimators, and the second and third terms regularize their total variations.

Conclusion

Although there have been some image restoration procedures proposed in the literature, this problem is far from being solved satisfactorily. For image denoising, most existing methods can preserve edges to certain degree. But some important edge structures, such as angles, corners, and places where edges have large curvature, would be blurred or rounded by them. So, edge-structure-preserving image denoising should be an interesting future research topic. For image deblurring, most existing methods assume that the psf is the same in the entire observed image, which may not be true in certain applications. Image deblurring with variable psf should be an important topic for future research, although it is technically challenging.

The current research on image restoration is often *ad hoc* in nature. People usually suggest their methods based on intuition and justify their methods by certain numerical comparisons with existing ones. In which situations would a suggested method work? Where in a given image would the method restore the true image well and where would it perform poorly? How should we choose the associated procedure parameters automatically based on observed data? We usually do not have answers to these important questions, which requires much future research effort.

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Cross-References

Computer vision, Image processing, Nonparametric regression