

## Sample Final Examination

This was a closed book two-hour exam. Students were allowed *two* 8.5" by 11" sheet of notes. A book of tables was provided. This included standard normal probabilities,  $t$  distribution critical values,  $\chi^2$  distribution critical values, and F distribution critical values. Most of the MacAnova output was in a separate exhibit booklet

1. During the 1996 presidential campaign, one of the Republican candidates based his campaign on a proposal for a "flat tax" (income tax with just one rate).

A polling organization interviewed random samples of Republicans in Delaware and Arizona, both on the same day. In both states those interviewed were asked the identical question "Do you favor the flat tax proposal?". Out of 260 Delaware Republicans interviewed, 90 favored the flat tax and 170 did not, while out of 140 Arizona Republicans interviewed, 62 favored the flat tax and 78 did not.

(a) Is there evidence the actual proportion of Delaware Republicans favoring the flat tax differs from the actual proportion of Arizona Republicans that favor it? Use  $\alpha = .05$

(b) Find a 95% confidence interval for the difference between Delaware and Arizona in the actual proportions of Republicans favoring the flat tax.

(c) Suppose now the actual proportion of Arizona Republicans who favor the flat tax is .45. How many Arizona Republicans must be sampled for the 95% margin of error in estimating this proportion to be 4% (.04)?

2. University researchers put 12 SafeHouse brand radon detectors in a chamber that exposed them to 105 picocuries per liter of radon. The readings were as follows

91.9	97.8	111.4	122.3	105.4	95.0
103.8	99.6	96.6	119.3	104.8	101.7

**Exhibit 2** (Problem 2)

```
Cmd> radon <- vector(91.9,97.8,111.4,122.3,105.4,95,103.8,\
  99.6,96.6,119.3,104.8,101.7)
```

```
Cmd> sum(radon)
(1)      1249.6
```

```
Cmd> sum((radon - sum(radon)/12)^2)
(1)      971.43
```

(a) Is there convincing evidence that SafeHouse brand radon detectors are biased, that is their mean measurement differs from 105? Use  $\alpha = .05$ . Be sure to state the null and alternative hypotheses.

(b) Find a 95% confidence interval for the mean measurement of SafeHouse brand radon detectors when exposed to 105 picocuries per liter of radon.

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3. Two astronomers, H and M measured the corrected red shift (km/sec) of 10 type SO galaxies. Here is a table of their results:

		Galaxy Number									
Astronomer		1	2	3	4	5	6	7	8	9	10
H	$x_1$	1507	858	1205	832	2206	924	4607	2592	1930	5378
M	$x_2$	1471	778	1155	915	2194	1033	4430	2670	2050	5278
Difference	$d$	36	80	50	-83	12	-109	177	-78	-120	100

**Exhibit 3**

```

Cmd> x1 <- vector(1507,858,1205,832,2206,924,4607,2592,1930,5378)
Cmd> x2 <- vector(1471,778,1155,915,2194,1033,4430,2670,2050,5278)
Cmd> d <- x1 - x2
Cmd> vector(sum(x1),sum(x2),sum(d))
(1)          22039          21974          65
Cmd> xbar1 <- describe(x1,mean:T); xbar2 <- describe(x2,mean:T)
Cmd> write(vector(sum((x1 - xbar1)^2),sum((x2 - xbar2)^2)))
VECTOR:
(1)          22890618.9          21349016.4
Cmd> write(sum((x1 - xbar1)*(x2 - xbar2)))
NUMBER:
(1)          22074567.4
Cmd> dbar <- describe(d,mean:T)
Cmd> write(sum((d - dbar)^2))
NUMBER:
(1)          90500.5

```

(a) Is this a paired or a two sample comparison? Explain your answer.

*For the remainder of this question, your answers must be consistent with your answer to (a).*

(b) H was a fairly inexperienced graduate student and M was an experienced professor. Is there evidence at the 10% level of significance that H's measurements tended to be systematically greater than those of M.

(c) Find a 95% confidence interval for the difference between the average of measurements by H and the average of measurements by M?

4. The Cheese data set in the Appendix to *IPS* contains data relating the taste of cheese to the concentrations of three chemicals, acetic acid, hydrogen sulfide and lactic acid. The variables are

- taste Taste score, an average of scores from several tasters
- acetic natural log of the concentration of acetic acid
- h2s natural log of the concentration of hydrogen sulfide
- lactic concentration of lactic acid

There are 30 samples of cheese, all manufactured by the same process.

**Exhibit 4**

```

Cmd> readdata("") # file cheese.txt
# File cheese.txt from publisher's web site
# Data for Appendix data set CHEESE, p. D-1 of IPS4
# Col. 1: id = case number (1-30)
# Col. 2: taste = taste score combined from several tasters
# Col. 3: acetic = log acetic acid concentration
# Col. 4: h2s = log hydrogen sulfide concentration
# Col. 5: lactic = lactic acid concentration
Read from file "TP1:Stat5021:Stat5021S03:Data:Appendix:cheese.txt"
Column 1 saved as REAL vector id
Column 2 saved as REAL vector taste
Column 3 saved as REAL vector acetic
Column 4 saved as REAL vector h2s
Column 5 saved as REAL vector lactic

Cmd> regress("taste=acetic+h2s+lactic",pval:T)
Model used is taste=acetic+h2s+lactic

```

	Coef	StdErr	t	P-Value
CONSTANT	-28.877	19.735	-1.4632	0.1554
acetic	0.32774	4.4598	0.073489	0.94198
h2s	3.9118	1.2484	3.1334	0.0042471
lactic	19.671	8.6291	2.2796	0.031079

```

N: 30, MSE: 102.63, DF: 26, R^2: 0.65177
Regression F(3,26): 16.221, P-value: 3.8102e-06, Durbin-Watson:
1.5751
To see the ANOVA table type 'anova()'

Cmd> SS

```

	CONSTANT	acetic	h2s	lactic	ERROR1
	18057	2314.1	2147	533.32	2668.4

```

Cmd> DF

```

	CONSTANT	acetic	h2s	lactic	ERROR1
	1	1	1	1	26

```

Cmd> regpred(vector(6,8,1.6),estimate:F)# acetic=6,h2s=8,lactic=1.6
component: SEest
(1) 2.7302
component: SEpred
(1) 10.492

```

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(a) What is the null hypothesis  $H_0$  tested by  $F = 16.221$  (underlined in `regress()` output)? What is the alternative hypothesis  $H_a$ ? What critical value should be used for a test when  $\alpha = .01$ ?

(b) Find a 95% confidence interval for the coefficient of `lactic`.

(c) Estimate with a 95% confidence interval the mean taste score when log acetic acid concentration is 6, log hydrogen sulfide concentration is 8, and lactic acid concentration is 1.6.

5. An experiment was conducted to compare birch plywood made with 6 different resin glues, numbered here 1 through 6. 10 pieces of plywood made with each glue were tested and the shear strength (PSI = pounds per square inch) measured. In the following table are the values of PSI for each of the 60 pieces of plywood.

Glue	1	2	3	4	5	6
Mean shear strength	502	470	500	520	551	620
	458	483	502	510	556	643
	445	478	480	582	592	589
	479	493	519	530	562	576
	468	498	459	495	566	576
	463	466	499	543	558	581
	517	492	500	513	573	606
	494	479	509	540	557	633
	499	534	528	523	633	562
	463	531	538	532	638	579

**Exhibit 5**

```

Cmd> psi <- vector(502,458,445,479,468,463,517,494,499,463,\
  470,483,478,493,498,466,492,479,534,531,\
  500,502,480,519,459,499,500,509,528,538,\
  520,510,582,530,495,543,513,540,523,532,\
  551,556,592,562,566,558,573,557,633,638,620,\
  643,589,576,576,581,606,633,562,579)

Cmd> glue <- factor(vector(rep(1,10),rep(2,10),rep(3,10),\
  rep(4,10),rep(5,10),rep(6,10)))

Cmd> print(format:"1.0f",glue)
glue:
(1) 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3
(31) 4 4 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 6

```

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```

Cmd> tabs(psi,glue,mean:T,count:T,stddev:T)
component: mean
(1)      478.8      492.4      503.4      528.8      578.6
(6)      596.5
component: count
(1)       10       10       10       10       10
(6)       10
component: stddev
(1)      23.16      23.396      22.707      23.63      32.139
(6)      27.428
    
```

```

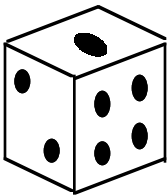
Cmd> anova("psi=glue")
Model used is psi=glue

          DF          SS          MS
CONSTANT    1  16838104  16838104
glue         5   115280   23056.1
ERROR1      54   35486.9   657.165
    
```

(a) On the basis of these data, can you statistically conclude that the six glues differ with respect to the PSI of plywood made using them? Use  $\alpha = .05$ . Be sure to give the critical value used.

(b) Glues 1 and 3 were made by different manufacturers and were intended to have the same properties. Is there statistical evidence at the 5% level of significance that their mean strengths were not the same?

6.



A die is a cube whose faces have 1, 2, 3, 4, 5, or 6 spots with 6 opposite 1, 5 opposite 2, and 4 opposite 3. When a fair die is rolled, each face should have equal probability  $1/6$  of being on top and hence each number should have probability  $1/6$ . A particular die has its spots deeply cut into the surface. It is conjectured that it is biased in favor of higher numbers,

because lower numbers have less material cut away and are thus more likely to be on the bottom. If the die is not biased, a 5 or a 6 should occur with probability  $p = 2/6 = 1/3$ . If the die is biased as conjectured then  $p > 1/3$ .

The following experiment procedure is proposed to examine this conjecture.

The die is to be rolled independently 7 times and the number of times (X) a 5 or a 6 is on top will be counted. If  $X \geq 5$  (that is, at least five 5's or 6's appeared out of 7 trials), the die will be considered to be biased.

Here is an table of the probability distributions of X when  $p = 1/3$  and when  $p = 3/4$ .

X	0	1	2	3	4	5	6	7
$p = 1/3$	0.0585	0.2048	0.3073	0.2561	0.1280	0.0384	0.0064	0.0005
$p = 3/4$	0.0001	0.0013	0.0115	0.0577	0.1730	0.3115	0.3115	0.1335

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- (a) The procedure described can be considered to be a test of a particular null hypothesis  $H_0$  against a particular alternative hypothesis  $H_a$ . State  $H_0$  and  $H_a$ .
- (b) Find the *exact* value of the significance level  $\alpha$  for this test. *Do not use a normal approximation.*
- (c) Suppose in fact that  $p = 3/4$ . Find the *exact* value of  $\beta$  for this test. *Do not use a normal approximation.*