1. The mean of X is

$$E(X) = \int x f(x) dx = \int_a^b \frac{x}{b-a} dx = \frac{b^2 - a^2}{2(b-a)} = \frac{a+b}{2}.$$

2. $E(X) = \frac{1}{100}(1 + 2 + \cdots + 100) = \frac{1}{100}\frac{(100)(101)}{2} = 50.5.$

8. $E(XY) = \int_0^1 \int_0^x xy \cdot 12y^2 dy dx = \frac{1}{2}.$

[illegible]

11. The p.d.f.'s of Y_1 and Y_n were found in Sec. 3.9. For the given uniform distribution, the p.d.f. of Y_1 is

$$g_1(y) = \begin{cases} n(1-y)^{n-1} & \text{for } 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore,

$$E(Y_1) = \int_0^1 yn(1-y)^{n-1} dy = \frac{1}{n+1}.$$

The p.d.f. of Y_n is

$$g_n(y) = \begin{cases} ny^{n-1} & \text{for } 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore,

$$E(Y_n) = \int_0^1 y \cdot ny^{n-1} dy = \frac{n}{n+1}.$$

2. $E(2X_1 - 3X_2 + X_3 - 4) = 2E(X_1) - 3E(X_2) + E(X_3) - 4 = 2(5) - 3(5) + 5 - 4 = -4.$

6. Let $X_i = 1$ if the ith jump of the particle is one unit to the right and let $X_i = -1$ if the ith jump is one unit to the left. Then, for $i = 1, \ldots, n$,

$$E(X_i) = (-1)p + (1)(1-p) = 1 - 2p.$$

The position of the particle after n jumps is $X_1 + \cdots + X_n$, and

$$E(X_1 + \cdots + X_n) = E(X_1) + \cdots + E(X_n) = n(1 - 2p).$$

8. It follows from Example 4.2.4 that

$$E(X) = 8\left(\frac{10}{25}\right) = \frac{16}{5}.$$

Since $Y = 8 - X$, $E(Y) = 8 - E(X) = \frac{24}{5}$. Finally, $E(X - Y) = E(X) - E(Y) = -\frac{8}{5}$.

11. We shall use the notation presented in the hint for this exercise. It follows from part (a) of Exercise 10 that $E(X_i) = 2$ for $i = 1, \ldots, k$. Therefore,

$$E(X) = E(X_1) + \cdots + E(X_k) = 2k.$$