11. This exercise is similar to exercise 10. Let the sample space consist of all subsets (unordered) of 12 out of the 100 people in the group. There are $\binom{100}{12}$ such subsets. The number of subsets that contain A and B is the number of subsets of size 10 out of the other 98 people, $\binom{98}{10}$, so the probability we want is

$$\frac{\binom{98}{10}}{\binom{100}{12}} = \frac{12 \times 11}{100 \times 99} = 0.01333.$$

13. This exercise is similar to Exercise 12. Here, we want four designated bulbs to be in the same group. The probability is

$$\frac{\binom{20}{6} + \binom{20}{10}}{\binom{24}{10}} = 0.1140.$$

15. (a) If we express 2^n as $(1+1)^n$ and expand $(1+1)^n$ by the binomial theorem, we obtain the desired result.

    (b) If we express 0 as $(1-1)^n$ and expand $(1-1)^n$ by the binomial theorem, we obtain the desired result.

2. We are asked for the number of arrangements of four distinct types of objects with 18 or one type, 12 of the next, 8 of the next and 12 of the last. This is the multinomial coefficient $\binom{50}{18, 12, 8, 12}$.

4. There are $\binom{10}{3, 3, 2, 1, 1}$ arrangements of the 10 letters of four distinct types. All of them are equally likely, and only one spells statistics. So, the probability is $1/\binom{10}{3, 3, 2, 1, 1} = 1/50400$.

8. There are $\binom{52}{13, 13, 13, 13}$ ways of distributing the cards to the four players. There are $\binom{12}{3, 3, 3, 3}$ ways of distributing the 12 picture cards so that each player gets three. No matter which of these ways we choose, there are $\binom{40}{10, 10, 10, 10}$ ways to distribute the remaining 40 nonpicture cards so that each player gets 10. So, the probability we need is

$$\frac{\binom{12}{3, 3, 3, 3}\binom{40}{10, 10, 10, 10}}{\binom{52}{13, 13, 13, 13}} = \frac{\frac{12!}{(3!)^4}\frac{40!}{(10!)^4}}{\frac{52!}{(13!)^4}} \approx 0.0324.$$

2. Let A, B, and C stand for the events that a randomly selected family subscribes to the newspaper with the same name. Then $\Pr(A \cup B \cup C)$ is the proportion of families that subscribe to at least one newspaper. According to Theorem 1.10.1, we can express this probability as

$$\Pr(A) + \Pr(B) + \Pr(C) - \Pr(AB) - \Pr(AC) - \Pr(BC) + \Pr(ABC).$$

The probabilities in this expression are the proportions of families that subscribe to the various combinations. These proportions are all stated in the exercise, so the formula yields

$$\Pr(A \cup B \cup C) = 0.6 + 0.4 + 0.3 - 0.2 - 0.1 - 0.2 + 0.05 = 0.85.$$

5. Determine first the probability that at least one guest will receive the proper hat. This probability is the value p_n specified in the matching problem, with $n = 4$, namely

$$p_4 = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} = \frac{5}{8}.$$

So, the probability that no guest receives the proper hat is $1 - 5/8 = 3/8$.

8. It is impossible to place exactly $n-1$ letters in the correct envelopes, because if $n-1$ letters are placed correctly, then the nth letter must also be placed correctly.