3. To prove the first result, let $x \in (A \cup B)^c$. This means that x is not in $A \cup B$. In other words, x is neither in A nor in B. Hence $x \in A^c$ and $x \in B^c$. So $x \in A^c \cap B^c$. This proves that $(A \cup B)^c \subset A^c \cap B^c$. Next, suppose that $x \in A^c \cap B^c$. Then $x \in A^c$ and $x \in B^c$. So x is neither in A nor in B, so it can't be in $A \cup B$. Hence $x \in (A \cup B)^c$. This shows that $A^c \cap B^c \subset (A \cup B)^c$. The second result follows from the first by applying the first result to A^c and B^c and then taking complements of both sides.

7. (a) These are the points not in A, hence they must be either below 1 or above 5. That is $A^c = \{x : x < 1 \text{ or } x > 5\}$.
   (b) These are the points in either A or B or both. So they must be between 1 and 5 or between 3 and 7. That is, $A \cup B = \{x : 1 \le x \le 7\}$.
   (c) These are the points in B but not in C. That is $BC^c = \{x : 3 < x \le 7\}$. (Note that $B \subset C^c$.)
   (d) These are the points in none of the three sets, namely $A^cB^cC^c = \{x : 0 < x < 1 \text{ or } x > 7\}$.
   (e) These are the points in the answer to part (b) and in C. There are no such values and $(A \cup B)C = \emptyset$.

3. (a) If A and B are disjoint then $B \subset A^c$ and $BA^c = B$, so $\Pr(BA^c) = \Pr(B) = 1/2$.
   (b) If $A \subset B$, then $B = A \cup (BA^c)$ with A and BA^c disjoint. So $\Pr(B) = \Pr(A) + \Pr(BA^c)$. That is, $1/2 = 1/3 + \Pr(BA^c)$, so $\Pr(BA^c) = 1/6$.
   (c) According to Exercise 4 in Section 1.4, $B = (BA) \cup (BA^c)$. Also, BA and BA^c are disjoint so, $\Pr(B) = \Pr(BA) + \Pr(BA^c)$. That is, $1/2 = 1/8 + \Pr(BA^c)$, so $\Pr(BA^c) = 3/8$.

12. The events $B_1, B_2, \ldots$ are disjoint, because the event B_1 contains the points in A_1, the event B_2 contains the points in A_2 but not in A_1, the event B_3 contains the points in A_3 but not in A_1 or A_2, etc. By this same reasoning, it is seen that $\cup_{i=1}^n A_i = \cup_{i=1}^n B_i$ and $\cup_{i=1}^\infty A_i = \cup_{i=1}^\infty B_i$. Therefore,

$$\Pr\left(\bigcup_{i=1}^n A_i\right) = \Pr\left(\bigcup_{i=1}^n B_i\right)$$

and

$$\Pr\left(\bigcup_{i=1}^\infty A_i\right) = \Pr\left(\bigcup_{i=1}^\infty B_i\right).$$

However, since the events $B_1, B_2, \ldots$ are disjoint,

$$\Pr\left(\bigcup_{i=1}^n B_i\right) = \sum_{i=1}^n Pr(B_i)$$

and

$$\Pr\left(\bigcup_{i=1}^\infty B_i\right) = \sum_{i=1}^\infty \Pr(B_i).$$

13. We know from Exercise 12 that

$$\Pr\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n \Pr(B_i).$$

Furthermore, from the definition of the events $B_1, B_2, \ldots$, it is seen that $B_i \subset A_i$ for $i = 1, \ldots, n$. Therefore, by Theorem 1.5.4, $\Pr(B_i) \le \Pr(A_i)$ for $i = 1, \ldots, n$. It now follows that

$$\Pr\left(\bigcup_{i=1}^n A_i\right) \le \sum_{i=1}^n \Pr(A_i).$$

Of course, if the events $A_1, \ldots, A_n$ are disjoint, there is equality in this relation.

3. This is a simple matter of permutations of five distinct items, so there are $5! = 120$ ways.

7. There are 20^{12} possible outcomes in the sample space. If the 12 balls are to be thrown into different boxes, the first ball can be thrown into any one of the 20 boxes, the second ball can then be thrown into any one of the other 19 boxes, etc. Thus, there are $20 \cdot 19 \cdot 18 \cdots 9$ possible outcomes in the event. So the probability is $20!/[8!20^{12}]$.

10. We can imagine that the 100 balls are randomly ordered in a list, and then drawn in that order. Thus, the required probability in part (a), (b), or (c) of this exercise is simply the probability that the first, fiftieth, or last ball in the list is red. Each of these probabilities is the same $\frac{r}{100}$, because of the random order of the list.