An Improved Award System for Soccer

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May 2002

Historically, say prior to 1980, soccer leagues awarded two points to the winning team in a match and zero points to the losing team. In the case of a tie each team was awarded one point. The final standings for a league was based on the total number of points earned by each team during the season. There was concern with the lack of goal scoring under that system and with the proportion of games which end in a tie. In part these outcomes result from the weaker team playing a very conservative or defensive strategy. To try to overcome these problems several important European leagues replaced the 2-1-0 award system with the 3-1-0 award system where the winning team was awarded three points instead of two. The 3-1-0 system is now common throughout the world including the quadrennial World Cup competition.

England was the pioneer in adopting the 3-1-0 award system. It was first used there in the 1981/82 season. In the following years it was taken up in other countries. For ten different countries we collected data from leading leagues which changed from the 2-1-0 award system to the 3-1-0 award system. For example for England we collected data for the 17 seasons preceding 1981/82 for the 2-1-0 award system and the subsequent 19 seasons under the 3-1-0 award system. In the ten countries for the data collected the smallest number of games played under one of the two award systems was

*Research supported in part by NSF Grant DMS 9971331

1,230 while the largest number was 9,616. For each of the ten countries we calculated the average number of goals scored per game and the proportion of ties under the two award systems. The results are given in the Table 1.

From the table we see that there is little evidence to support either the hypothesis that the 3-1-0 award system led to increased scoring or the hypothesis that it led to fewer ties. Half the countries did have increased scoring after the change but the others had a decrease in scoring. Six of the ten had fewer ties but only Turkey and Italy saw noteworthy decreases. Here we will explain why the switch to the 3-1-0 award system did not have the desired results and propose a new award system that should increase scoring.

Place Table 1 about here

Computing expected winnings

The main tool that we use to discuss team strategy is to compute a team's expected winnings, that is the number of points they expect to earn from a game. Suppose that Barcelona and Madrid are to play a game where we know that on the average Barcelona scores λ (2 in our example) goals per game and Madrid scores μ (1.5 in our example) goals per game. Several researchers have found that the distribution of scores obtained by each team can be well approximated by the Poisson distribution (see sidebar). If we let B and M represent the number of goals each will score then a reasonable model for the final score is to assume that B and M have independent Poisson distributions with mean parameters λ and μ respectively. With these assumptions we can compute the probability for any possible outcome of the game, e.g., the probability that Barcelona wins 2-1 or the probability that Madrid wins 3-0. This is done using the Poisson distribution; more details are provided in the sidebar. By considering all possible scores we can evaluate the probability that each team wins and that the teams play to a draw. Hence before the game begins we can compute the expected winnings for each team. Under the 2-1-0 award system Barcelona's expected winnings are just two times the probability that Barcelona scores more goals than Madrid plus one times the probability they score the same number of goals. The answer just depends on the values of λ and μ. With $\lambda = 2$ and $\mu = 1.5$, Barcelona wins with probability .49, loses with probability .29 and draws with probability .22; the expected number of league points earned by Barcelona is $0.49 \times 2 + 0.22 = 1.20$.

Suppose that at the end of the first half neither team has scored. Now if Barcelona scores on the average λ goals per game we would expect that on the average it would score $\lambda/2$ goals in the second half. Similarly we would expect Madrid to score $\mu/2$ goals in the second half. Hence assuming that there was no score in the first half we can compute the expected winnings of each team at the beginning of the second half. As we shall soon see their expected winnings at half time will be different from their expected winnings at the start of the game.

More generally if we let $0 \leq t \leq 1$ represent the proportion of the game that has been played. If neither team has scored by time t then under our Poisson model the number of goals that Barcelona and Madrid will score in the remainder of the game follow independent Poisson distributions with mean parameters $\lambda(1-t)$ and $\mu(1-t)$ respectively. Hence we can compute the expected winnings for each team at this point of the game. In much the same way we can calculate the expected winnings of Barcelona if it is ahead by one goal or two goals at time t or behind by one goal or two goals and so on. The same is true for Madrid. However under the 2-1-0 award system at any stage of the game one only needs to compute the expected winnings of one of the teams since the sum of their expected winnings must always be two (because two points are allocated to the two teams in each game).

As an example we calculated the expected winnings of Barcelona over the course of the game under two different scenarios. One where the score is tied and the other when they are ahead by one. In each case we assumed that on the average Barcelona scores 2.0 goals per game and Madrid scores 1.5 goals. The results are given in Figure 1. Note that as the game progresses Barcelona's expected winnings decrease from 1.2 at the beginning to 1.0 at the end when the score is tied. If Barcelona is ahead by one goal, the expected winnings increase from 1.6 (if they score early in the match) to 2.0 (if they lead when the match ends). This is typical under the 2-1-0 award system. The stronger team's expected winnings always decrease when the score is tied or they are behind and increases when they are ahead.

Place Figure 1 about here

So far we have assumed that the 2-1-0 award system was being used. But similar calculations can be done for the 3-1-0 award system. In fact for any award system we can compute the expected winnings of either team at any stage of the game as long as we know the two teams original Poisson parameters, the time remaining and the current score. Next we will study teams' expected winnings under both the 2-1-0 and 3-1-0 award systems.

Strategy under the standard systems

We begin with the 2-1-0 award system and consider the behavior of the expected winnings of Barcelona as the game progresses. We first consider the situation where the score is tied. In Figure 1 we saw that when $\lambda = 2.0$ and $\mu = 1.5$ Barcelona's expected winnings always decrease as t moves from zero to one. This remains true whenever Barcelona is the better team that is when $\lambda > \mu$. Hence, the longer the game goes on without a goal being scored the more disadvantageous it becomes for the better team. Conversely Madrid's expected winnings are increasing as the game progresses. The longer the game continues without a score the more advantageous it becomes for the weaker team. This helps to explain the popular strategy of the weaker team playing very conservatively from the beginning of the game. By playing very defensively the weaker team is decreasing the scoring rates for both teams to values closer to zero. When both of the Poisson mean parameters are quite small both teams have expected winnings close to one. This is better for the weaker team than playing normally. Hence playing defensively or conservatively is in effect shortening the game. This makes sense intuitively since the shorter the game the less chance the better team has to demonstrate its superiority.

More generally anytime a team's expected winnings are increasing as the game progresses then it should use a defensive or conservative strategy which limits goal scoring and in effect shortens the game. On the other hand if a team's expected winnings are decreasing with time it should continue to play normally. It could even consider using a more aggressive or offensive strategy which increases goal scoring and in effect lengthens the game. This observation is basic to understanding how a team should determine their style of play during the game. In order to maximize their payoff or expected winnings they should play defensively when their expected winnings are increasing and play normally when their expected winnings are decreasing. This underlies much of what we say in the following.

Next we consider the situation where Barcelona, the stronger team, has scored first. When $\lambda = 2.0$ and $\mu = 1.5$ we saw in Figure 1 that their expected payoff was increasing as t moves up to one. That is once the stronger team gets ahead it is in their interest to play defensively, decrease both teams chances of scoring and in effect shorten the game. On the other hand the weaker team should play more aggressively once they get behind.

Suppose the weaker team happens to score first. Then calculations demonstrate, as one would expect, that it is in their interest to play defensively and decrease the likelihood of any goal being scored. All these results show that no matter what the state of the game it is always in the interest of one of the teams to play conservatively except when they are of equal strength and the score is tied. This helps to explain the lack of scoring under the 2-1-0 award system.

In the 3-1-0 award system the sum of the expected winnings for the two teams is not constant as it is for the 2-1-0 award system. However calculations reveal that the optimal strategies for the two teams are the same as in the 2-1-0 award system. The weaker team should play conservatively until it gets behind. The stronger team should play normally until it gets ahead. Once either team gets ahead they should play conservatively and the opponent aggressively.

A new award system

The problem with the standard systems is that no matter what the state of game and the relative strengths of the teams it is always in one of their best interest to try to limit the likelihood of a goal. The only exception is when the teams are of equal ability and the score is tied. In light of this we wish to find an award system which encourages both to play normally throughout a portion of the game. As a practical matter any new system should not be too different from the standard systems.

We now propose such an award system. In this system each team receives zero points for a tie. If a team wins by three or more goals it receives three points. If it wins by one or two goals it receives one point. The losing team receives -1.5 points if it loses by three or more goals and receives -0.5 points if it loses by one or two goals. The payoffs for the three systems are summarized in Table 2. Under this system at the end of the year teams near the bottom of the standings would have a negative point total for the season. Just as with the 3-1-0 award system, we have with the new system that the sum of the expected winnings is not constant and the sum of the total payoffs for any game depends on its outcome. But as before one can compute the expected winnings for either team at any point in the game.

Place Table 2 about here

Strategy under the new system

To see how the new system could affect teams' strategies we need to find the optimal strategy of each team for different game situations and different choices of the Poisson parameters. We return to the example discussed earlier where Barcelona scores on the average $\lambda = 2.0$ goals per game and Madrid $\mu = 1.5$ goals.

First we assume that no goals have been scored and we plot the expected winnings of each team as the game progresses. This is graph I in Figure 2. The upper curve is Barcelona's expected winnings which decreases as the game continues with no score. No surprise here as Barcelona is the stronger team. The decreasing expected winnings means that Barcelona will continue to play its normal style. Note however that Madrid's expected winnings (the lower curve) increases until about 25% of the game remains and then they start to decrease. Recall that under the old award system Madrid's expected winnings increased throughout the game under this scenario. Hence Madrid's optimal strategy under the proposed award system is to play conservatively until about one-fourth of the game remains and then play normally. What is the reason for this welcome change? When there is only a small part of the game left it is most likely that no goals or just one goal will be scored in the remainder of the game. When the skill levels of the two teams are not too different the probability that the weaker team scores first, although less than one-half, will be close enough to one-half that it becomes in their interest to play for a win because the reward for winning by one goal (1 point) is twice the penalty for losing by one goal (-0.5 points). If the weaker team's probability of scoring first is greater than 1/3 it should stop playing conservatively and go for the goal. The point at which the weaker team should switch from conservative to normal strategy depends on the relative strengths to the two teams. But when the two teams are not too different in ability the new scoring system encourages both to play for the win when the score is tied near the end of a game.

Next we assume that Barcelona is ahead by one goal. Under the standard award systems their optimal strategy would be to play conservatively for the rest of the game. We see in graph II of Figure 2 however that their expected winnings (the upper curve) continues to decrease until less than 20% of the game remains. So they should only switch to a conservative style near the end of the contest. Why is that? Since they are the better team it is in their interest to stay aggressive to try to win by three goals since they expect to score more goals than Madrid if both teams play their normal game. Only near the end should Barcelona play conservatively when it becomes unlikely that they can score two more goals before Madrid scores one. How near the end they should start to be conservative depends on the values of λ and μ. On the other hand Madrid should also continue to play normally because its expected winnings (the lower curve) in graph II are also decreasing. This is because losing by two is no worse than losing by one and they have a reasonable chance to score the next goal.

In graph III of Figure 2 we assume that Madrid is ahead by one goal. Their expected winnings (the upper curve here) is increasing over the entire range. Since they are the weaker team their optimal strategy is to forget about trying to play for a win by at least three goals and play defensively to conserve their one goal lead. Barcelona of course should continue to play normally.

In Figure 3 we assume that the two teams are equal with $\lambda = \mu = 2$ and that Barcelona is ahead by one goal. Madrid trails and should thus play normally. Barcelona's expected winnings (the upper curve) is decreasing until about 27% of the game remains; here again Barcelona plays normal at first to try for a big win and only turns conservative when the chances of a big win are small. Again the point at which the expected winning curve turns and the optimal strategy changes depends on the scoring rates. If each team only scores on the average one goal a game, then the first team to score will turn to conservative play earlier. Other game situations can be analyzed in a similar manner.

These examples suggest the following differences between the proposed new award system and the standard systems.

- Under the proposed new system when the score is tied the weaker team should switch from a conservative to a normal style of play near the end of the game. Under the standard systems the weaker team should always play conservatively.

- Under the proposed new system once the stronger team gets ahead by one goal it should continue to play normally until the later stages of the game. Under the standard system the stronger team should switch to a conservative strategy once it gets the lead.

- Under the proposed new system once a significantly weaker team gets behind by one goal it should play conservatively until near the end of the game. Under the standard systems it should always play normally once it falls behind.
- Under the proposed new system once the superior team gets ahead by two goals it should continue to play normally and the inferior team conservatively. (The teams reverse roles relative to the standard system because of the presence of a reward for big wins.)

In summary the two main advantages of the new system are that there can be significant portions of the game where both teams should play normally and except in the later portions of the game stronger team should continue to play normally once it gets ahead. The length of these intervals of normal play depends on the relative strengths of the two teams. But games where the better team continues to try to score until it is up by three goals should be more exciting than those where the better team becomes conservative after it gets ahead by one goal.

Final remarks

One possible objection to our approach and to the new award system is that teams likely do not have the ability or even an interest in mathematically computing their expect winnings. And even if they did, selecting the appropriate Poisson parameters for any given game would be very difficult. These depend on many factors including where the game is being played, the weather, the playing style of the opponent, the team's place in the league standings, the strategies of the coaches and on and on. In practice teams cannot gauge precisely their relatively strengths and we are not claiming that they always follow their optimal strategies especially when they are about evenly matched. But as we argued earlier the practice of the weaker team often playing for a tie and the negligible effect the change to the 3-1-0 award system seemed to have on the number of ties and the overall level of scoring suggests that teams can and do compute their expected winnings approximately when selecting their game strategies. We believe that this would remain true if the new system were adopted. In particular when the teams are roughly equal and not low scoring and the first goal is scored early in the game it is in neither of their interests to adopt a conservative style of play. Note that this state of affairs can be expected to hold in many actual games and no complicated computations need to be done for each team to decide that it is their best interests to continue to play normally. A real advantage of the new award system is that it encourages the better team to play normally throughout a great proportion of the game which should lead to more interesting games.

Another possible objection is that the proposal is too complicated. We considered some other award systems as well. They all awarded zero points for a tie, positive points for the winner and negative points for the loser where the points awarded could depend on the size of the win and the loss. It is easy to find others that work similarly to the one we have proposed. After considerable calculations we selected this one because it is not so different from the standard systems and we could not find another that seemed to work any better. Furthermore the new system is at most marginally more sophisticated than the 3-1-0 award system and it would take just a short time for both the teams and the fans to adapt. A related objection might be that the 3-1-0 system was also advertised as offering to provide more offense and failed to do so. Our approach here shows however that this is not surprising. Award systems that differentiate big wins and small wins are needed to actually provide an incentive for more attacking play during games.

The recent 2002 World Cup competition yields some suggestive evidence on this point. In the preliminary round the field of 32 teams were divided into eight groups of four. Within each group a round robin tournament was held to determine which two teams advanced to the final round. For teams with the same record the ties were broken by rules which depended on the number of goals scored. The final 16 teams then competed in a single elimination tournament. In the 15 elimination games in the second stage there were 26 goals scored for an average of 1.73 per game. Of these 2 came in overtime. Many commentators noted the paucity of scoring in these games. On the other hand in the 30 first round games involving at least one team that made it into the round of 16 there were 118 goals scored for an average of 3.93 per game. In the 6 first round games that involved two teams that made it into the final round of 16 there were 21 goals scored for an average of 3.5 goals per game. Clearly there was more scoring in the preliminary round than in the final round. One possible explanation is the presence of weaker teams in original field of 32. But more importantly, we believe, were the rules for breaking ties. In the preliminary round it was not necessarily in the interest of the better team to become conservative if they got the lead. While in the final round, where winning was the only thing that mattered, once a team got ahead it was clearly in their interest to become conservative.

Of the other major team sports hockey is the one that is most similar to soccer. This is true for how the teams interact during the game and in the determination of the standings. There are important differences however. In particular in hockey it is much more difficult for a weaker team to play defensively for a tie. This is because there are many more scoring opportunities in a typical hockey game than in a typical soccer game. One way to increase scoring in soccer would be to change the rules so that there would be more scoring opportunities. For example one could change the offside rule or increase the size of the goal. In contrast to American professional sports such as baseball, basketball and football, which are constantly tinkering with the rules in an attempt to produce a more entertaining product, soccer officialdom has been much more conservative about making changes. In lieu of making rule changes that affect how the game is played (eliminating offsides, changing the size of the goal), the award system we propose is the simplest way to achieve more entertaining games with more scoring.

Acknowledgments

We wish to thank Hal Stern for his many helpful comments during our writing of this piece.

Additional Reading

Keller, J. (1994). A characterization of the Poisson distribution and the probability of winning a game. *The American Statistician*, 48:294–298.

Lee, A. (1997). Modeling scores in the premier league: Is Manchester United really the best? *Chance*, 10:15–19.

	2-1-0 Award system			3-1-0 Award system		
Country	n_2	Average # of goals	Proportion of ties	n_3	Average # of goals	Proportion of ties
England	17	2.70	0.28	19	2.64	0.27
Iceland	20	2.93	0.26	16	2.97	0.22
N. Ireland	22	3.40	0.20	14	2.89	0.24
Norway	23	2.83	0.25	13	3.24	0.22
Turkey	23	2.01	0.34	13	2.85	0.24
Sweden	26	2.84	0.27	10	2.81	0.26
Finland	27	3.00	0.24	9	2.70	0.25
Italy	30	2.12	0.37	6	2.64	0.28
Spain	31	2.48	0.26	5	2.68	0.28
Germany	31	3.18	0.26	5	2.86	0.29

Table 1: The average number of goals scored per game and the proportion of games that end in tie for ten countries with n_2 seasons under the 2-1-0 award system and n_3 subsequent seasons under the 3-1-0 award system

Side bar Material–The Poisson Distribution in Soccer

The Poisson distribution models random events happening over time, for example goals in a soccer game. If $\lambda > 0$ is the average number of goals scored by team B during a typical game, then in a particular game the probability that team B scores k goals under the Poisson distribution is

$$Pr(B = k) = \frac{\exp^{-\lambda} \lambda^k}{k!}$$

If team M scores on the average $\mu > 0$ goals per game and the goal scoring of the two teams is independent, then the probability that B scores k goals and M scores l goals is

$$Pr(B = k, M = k) = \frac{\exp^{-\lambda} \lambda^k}{k!} \frac{\exp^{-\mu} \mu^l}{l!}$$

Though the assumption of two teams scoring independently can't be right since only one team can score at a time, the results of soccer games appear to be consistent with the independent Poisson model. Using these expressions it is possible to find the probability of any outcome. For example, with $\lambda = 2$

Game outcome	Probability
Barcelona wins by 5 or more	.019
Barcelona wins by 4	.036
Barcelona wins by 3	.081
Barcelona wins by 2	.149
Barcelona wins by 1	.209
Tie game	.216
Madrid wins by 1	.157
Madrid wins by 2	.084
Madrid wins by 3	.034
Madrid wins by 4	.011
Madrid wins by 5 or more	.004

Table 2: Probability of different outcomes under the Poisson model for Barcelona (2 goals per game) versus Madrid (1.5 goals per game)

and $\mu = 1.5$ (the case considered in the accompanying paper), the probability of a scoreless draw is .030, the probability of a 2-0 Barcelona win is .060, and the probability of a 0-2 Madrid win is .034, etc. By adding over all possible outcomes we find the distribution of outcomes for the game as follows

Game outcome	Points awarded		
	New	2-1-0	3-1-0
Win by ≥ 3	3	2	3
Win by 1 or 2	1	2	3
Tie	0	1	1
Lose by 1 or 2	-0.5	0	0
Lose by ≥ 3	-1.5	0	0

Table 3: The number of points awarded under the two standard systems and the new system for the various possible outcomes of a game

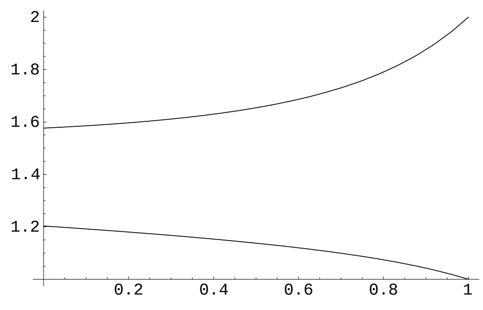
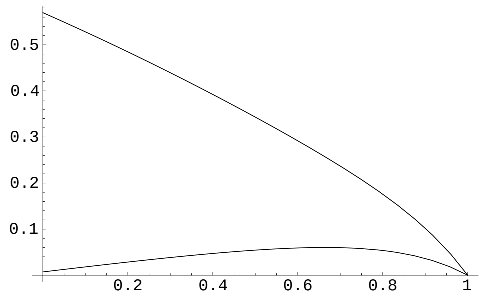
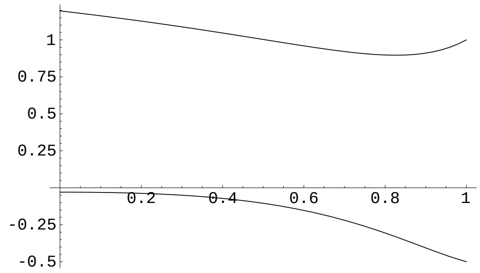


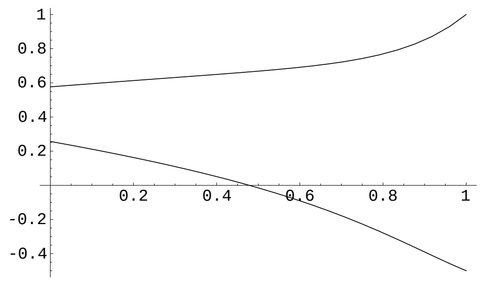
Figure 1: The expected winnings of Barcelona under the 2-1-0 award system over the duration of the game when they are ahead by one goal (the upper curve) and when the score is tied (the lower curve) where on the average they score 2.0 goals per game and Madrid scores 1.5 goals.



I: B(2.0) and M(1.5) tied



II: B(2.0) up one over M(1.5)



III: M(1.5) up one over B(2.0)

Figure 2: Over the duration of the game the expected winnings of Barcelona and Madrid under the new award system for three different situations. In graphs *I* and *II* the upper curve represents Barcelona's expected winnings and the lower curve Madrid's. In graph *III* Barcelona's is the lower curve and Madrid's the upper curve.

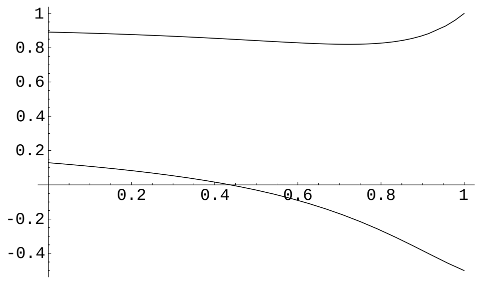


Figure 3: Over the duration of the game the expected winnings of Barcelona (upper curve) and Madrid (lower curve) under the new award system when Barcelona is ahead by one and they both score on the average two goals per game.