Problem 1

The sample percentage 53.2% is given in the problem statement. The first thing to do is calculate its SE using

$$SE \text{ for sample percent } = \frac{SD \text{ of box}}{\sqrt{\text{sample size}}} \times 100\%$$

where the SD of the box is estimated by the “short-cut formula”

$$SD \text{ of box } = \sqrt{\left( \text{fraction of 1s} \right) \times (\text{fraction of 0s})}$$

These formulas give

$$SD \text{ of box } = \sqrt{0.532 \times 0.468} = 0.499$$

and

$$SE \text{ for sample percent } = \frac{0.499}{\sqrt{1600}} \times 100\% = 1.2475\%$$

The EV for the sample percent is the true population percent, which is unknown but is assumed to be 50% in the null hypothesis. Thus the test statistic is

$$z = \frac{\text{sample percent} - \text{EV for sample percent}}{SE \text{ for sample percent}} = \frac{53.2\% - 50\%}{1.2475\%} = 2.565$$

Looking up 2.55 in the normal curve tail area table gives a $P$-value of 0.54% (less than 1% so “highly statistically significant”).

Problem 2

This is a two-sample test (the subject of Chapter 28). The fact that it is a controlled experiment does not make any difference (the subject of the latter half of Chapter 28). The test statistic is the difference of the two sample averages $7.15 - 6.88 = 0.27$ and which we standardize using

$$z = \frac{\text{test statistic} - \text{EV of test statistic}}{SE \text{ of test statistic}}$$

The EV of the test statistic is the difference of the population averages for treatment and control, which is assumed to be zero under the null hypothesis. Thus we only need to calculate the SE.

The first step is to calculate the SEs for the sample averages in the two groups using

$$SE \text{ for sample average } = \frac{SD \text{ of box}}{\sqrt{\text{sample size}}}$$
where we estimate the SD of the “error box” using the sample SD. So the SE for the treatment group is $0.81 / \sqrt{100} = 0.081$ and the SE for the control group is $0.87 / \sqrt{100} = 0.087$.

The next step calculates the SE for the difference of the two sample averages using the formula $\sqrt{a^2 + b^2}$ where $a$ and $b$ are the SEs for each average, that is, $\sqrt{0.081^2 + 0.087^2} = 0.11887$.

Now formula (1) becomes

$$z = \frac{0.27 - 0}{0.11887} = 2.27$$

Looking this $z$ up in the normal curve tail area table gives the $P$-value $P = 1.2\%$ (between 1.22 and 1.07, closer to the former).

**Problem 3**

(a) The $P$-value is not below 5%, so is not “statistically significant” by the convention the book (and almost everybody else) uses, and this means the difference is “just chance variation.” But the $P$-value is close to 5%, so it is hard to say anything one way or the other. It is a weak conclusion. The difference between the two poll results is almost, but not quite, large enough to be declared “statistically significant.” Candidate Smith may be gaining, but we can’t be sure.

(b) Two tails have twice the area of one tail, so the two-tailed $P$-value is $2 \times 5.7\% = 11.4\%$.

This part of the question didn’t ask for an interpretation, but $P = 11.4\%$ is nowhere near “statistically significant.” The conclusion “just chance variation” is now clear.

(c) The $P$-value is now well below 5%, so it is “statistically significant” and nearly “highly statistically significant” by the book’s convention. Now the conclusion is fairly strong. Candidate Smith is actually gaining.

**Problem 4**

To solve this problem, first we convert 650 to standard units, then look up in the normal curve tail area table.

650 is $650 - 305 = 145$ points above average, which is $145/110 = 1.32$ SDs. Thus 650 in standard units is +1.32.

Looking up in the table, the tail past $z = 1.32$ is about 9.3% (between 9.68 and 8.85 a bit closer to the former). 9.68 gets full credit, but 9.3% or 9.4% is a better answer.

**Problem 5**

The regression method says we

1. Convert the predictor variable (height) to standard units.
2. Multiply by $r$ giving the predicted response in standard units.
3. Convert back to original units of the response variable (weight).

**Step 1** 62 inches is $70 - 62 = 8$ inches below average. That’s $8/3 = 2.667$ SDs. Thus 62 inches is $-2.667$ in standard units (minus for below average).

**Step 2** $r \times (-2.667) = 0.47 \times (-2.667) = -1.253$ is the predicted weight in standard units.

**Step 3** $-1.253$ in standard units is $1.253$ SD below average. $1.253$ SD is $1.253 \times 30 = 37.6$ pounds, and $37.6$ pounds below average is $162 - 37.6 = 124.4$ pounds.

**Problem 6**

(a) The chance of a six in one roll is $1/6$. The chance in five rolls is

$$\left(\frac{1}{6}\right)^5 = \frac{1}{7776} = 0.00013$$

given by the multiplication rule.

(b) Very similar to part (a). The chance of a one or a two in one roll is $2/6 = 1/3$. The chance in five rolls is

$$\left(\frac{1}{3}\right)^5 = \frac{1}{243} = 0.0041$$

given by the multiplication rule.

(c) Here we need to use the pattern explained in the section on the “paradox of Chevalier de Mére” in the textbook.

We can’t do “chance of at least one six” directly. So we use the opposite rule and calculate the chance of the opposite event “no sixes.” The chance of no six on one roll is $5/6$. The chance in five rolls is

$$\left(\frac{5}{6}\right)^5 = \frac{3125}{7776} = 0.4019$$

or 40.2%. Thus the chance of the opposite event is $100\% - 40.2\% = 59.8\%$.

**Problem 7**

First we have to figure out the average and the SD of the box.

The average is

$$\frac{1 + 3 + 4 + 5 + 7}{5} = 4$$

The deviations from average are $-3, -1, 0, 1, 3$. So the SD is the RMS of this list

$$\sqrt{\frac{(-3)^2 + (-1)^2 + 0^2 + 1^2 + 3^2}{5}} = \sqrt{\frac{9 + 1 + 0 + 1 + 9}{5}} = 2$$
(a) The EV for the average of draws is the average of the box: 4.

(b) The SE is

\[ SE \text{ for average} = \frac{\text{SD of box}}{\sqrt{\text{number of draws}}} = \frac{2}{\sqrt{100}} = .2 \]

(c) To calculate this chance using the normal approximation we (1) convert 4.4 to standard units, call that \( z \), and (2) look up the tail area past \( z \) in the normal curve tail area table.

**Step 1** 4.4 is 0.4 above the EV, which is 2 SE above the EV, so 4.4 converted to standard units is +2.

**Step 2** The tail of the normal curve above +2 is 2.28%.

**Problem 8**

(a) The confidence interval is average of measurements ± 2 SE. The SE is estimated by

\[ \frac{\text{SD of measurements}}{\sqrt{\text{number of measurements}}} = \frac{0.23}{\sqrt{25}} = 0.046 \]

So 2 SE is 0.092 and the confidence interval is 511.13 ± 0.09.

(b) In two words you have to assume the “Gauss model” (Chapter 24 in the textbook).

Stated in more detail, you have to assume (1) there is no bias, (2) the measurements are statistically independent, (3) each measurement is a repetition of the same physical process.

Another way to say the same thing in probabilistic terms is that the measurements must be like draws with replacement from a box model and that the average of the box must be the true mass of the electron.

**Problem 9**

The first thing wrong is not following the advice of the box on p. 548 “report the \( P \)-value instead of just comparing \( P \) to 5%”.

The second thing wrong was data snooping. With 10 tests done, the “\( P \)-values are hard to interpret” as the book says in the box on p. 549. It is not clear that anything here is “statistically significant” especially since the \( P \)-values themselves were not reported. The whole study should just be ignored.