Statistics 5303 - Final Exam Sketched Solutions
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For questions 1 through 3, describe the experimental design that you would choose for each situation. Tell me about units, treatments, blocks, etc.
(1) Some chemicals may migrate from polystyrene ("styrofoam") cups used to serve hot coffee into the coffee. We will measure the concentration of toluene in hot water solutions after they have sat in polystyrene cups. Four factors are of interest: maker of cup (brands A and B), temperature of water (80 and 90 degrees C), length of time the water sits in the cup before measuring the response ( 15 minutes and 30 minutes), and pH of the water solution (6 and 6.5). We have resources for 48 measurements. There are no restrictions on the order in which the measurements should be done, but we can only make eight measurements before the equipment needs to be cleaned and recalibrated.

This is a $2^{4}$ confounded into blocks of size 8 . Because we have resources for 3 replications, we can use partial confounding. How about confounding $\mathrm{ABCD}, \mathrm{ABC}$, and ABD in the three replications.
(2) Commercial brownie mixes should be enhanced with nuts, M\&M's, and white chocolate chips to improve taste. Our current approach uses $1 / 4$ cup of each in a pan of brownies, but we want the perfect brownie! Design an experiment to determine the optimum additions. You may assume that the optimum lies between 0 and $1 / 2$ cup for each of the three factors. There is only one rater (me! though others can fight for the scraps after I'm full), and you may make up to 20 pans of brownies.

We'll need to fit a second order model to find an optimum, so let's use a central composite design centered at the current settings of $1 / 4$ cup each with a unit step of $1 / 8$ cup. We have 8 factorial points, 6 axial points, and 6 center points.
(3) Genetically modified bacteria may be able to produce insulin as they grow and divide. We wish to conduct a small scale experiment to study the effects of growth temperature and culture medium on the production of insulin. The bacteria will grow in a nutrient broth (the medium) in a large beaker; temperature is controlled by putting the beaker in an environmental chamber that maintains a uniform temperature. The environmental chamber is large enough for four beakers. There are three temperatures and four media we wish to study. We have resources to run 48 beakers.

Use a split plot. Temperature is the whole plot factor, and media is the split plot factor. Each environmental chamber run is a whole plot and each spot within the environmental chamber is a split plot. For each of questions 4-7, describe the design and give a skeleton anova.
(4) A common stereotype is that "beautiful is good;" that is, attractive people are stereotyped as having good personality qualities and unattractive people are stereotyped as having bad personality qualities. To test this hypothesis in employment screening, four ficticious and reasonably equivalent résumés are created (all are for women). We manipulate the "beautiful" nature of the résumés by including applicant pictures. We will use three pictures that have been rated as attractive, neutral, and unattractive, plus a control with no picture. These four résumés will be sent to six different personnel officers for initial screening for a job. For each personnel officer, the four résumés will be randomly assigned to the four picture treatments. The response is the rating given by the officer to each applicant.

This is a randomized complete block. The units are the resumes, the treatments are the pictures, and the blocks are the raters.

| Source | DF |
| :--- | ---: |
| block | 5 |
| treatments | 3 |
| error | 15 |

(5) New apple varieties are being introduced; we need to figure out which varieties yield more and how
irrigation affects yields for these new varieties. Four commercial orchards have agreed to participate in our experiment evaluating three new apple varieties and three irrigation schedules. At each orchard, three adjacent .5 hectare plots are cleared and then each plot is randomly assigned to one of three new varieties. All trees are allowed to grow in a similar environment for eight years. Yields will be measured in years 9 , 10 , and 11. The three irrigation schedules are randomly assigned to years for each .5 hectare plot. However, we note that there could be year to year variation, so we restrict the randomization so that each irrigation schedule is used once in each year at each orchard.

This is a split plot. Orchards are blocks, plots are whole plots, and years for each plot are split plots. The whole plot factor is variety, and the split plot factor is irrigation. An unusual twist is there is also a Latin square-like restriction that each irrigation schedule is used once in each year at each orchard.

| Source | DF |
| :--- | ---: |
| block (orchard) | 2 |
| variety | 2 |
| whole plot error | 4 |
| year (block) | 2 |
| schedule | 2 |
| var.sched | 4 |
| split plot error | 10 |

(6) The Pollution Control Agency wishes to study the effect of an additional control mechanism on the cadmium concentration of the outflow from a municipal waste water treatment plant. Twelve weeks are randomly divided into two groups of six. In the first group, the treatment plant will be run as usual. In the second group, the new control mechanism will also be used. Measurements will be taken during each week and an average cadmium concentration in the outflow will be computed for each week. In addition, an average nickel concentration will also be measured, as nickel and cadmium are often found together in the waste stream (and the control mechanism should not affect nickel concentrations).

This is a completely randomized design with week as unit and mechanism as treatment. Cadmium concentration is the response and nickel concentration is a covariate.

| Source | DF |
| :--- | ---: |
| Nickel | 1 |
| mechanism | 1 |
| error | 9 |

(7) Snellingdale Mall uses a lot of cut Christmas trees (real, not artificial) as decoration during the Christmas season. These trees are placed in clusters of three at ten locations around the mall. One important issue to them is how tree species affects how long the trees will retain their needles. This year they run an experiment. They get six each of Frasier Fir, Balsam Fir, Scotch Pine, White Pine, and Blue Spruce; all trees are the same size. The trees are then randomly spread around the mall subject to the restriction that each combination of three species occurs at one of the decoration locations. The response measured is how long each tree lasts before it begins to drop an unacceptable number of needles.

This is a BIBD. Locations are blocks, and species are treatments.

| Source | DF |
| :--- | ---: |
| block | 9 |
| treatments | 4 |
| error | 16 |

(8) The following experiment was conducted to study whether and how plants adapt to environment. Three populations of the same plant species are available (MN, KS, and OK). Five plants from each population are chosen as males (total of 15 males). Four plants from each population are chosen for each male; these plants will be females (total of 60 females). Pollen from each male plant is used to fertilize the flowers on its assigned female plants. Three seeds are collected from each pollinated female (total 180 seeds). These three seeds are randomly assigned to one of three growth environments. The response is the height of each plant after 10 weeks of growth.

Construct a Hasse diagram for this design.


P population
S sire (population)
D dam (sire)
G growth environment
(9) Consider a $2^{4-1}$ factorial with $\mathrm{I}=-\mathrm{ABCD}$ as generator, blocked into two blocks of size four using $A B=-C D$.
(a) Give a skeleton ANOVA for this design.

| Source | DF |
| :--- | :--- |
| block $(\mathrm{AB}=-\mathrm{CD})$ | 1 |
| $\mathrm{~A}(=-\mathrm{BCD})$ | 1 |
| $\mathrm{~B}(=-\mathrm{ACD})$ | 1 |
| $\mathrm{C}(=-\mathrm{ABD})$ | 1 |
| $\mathrm{D}(=-\mathrm{ABC})$ | 1 |
| Error $(\mathrm{AC}=-\mathrm{BD}, \mathrm{AD}=-\mathrm{BC})$ | 2 |

(b) Tell which treatments are assigned to each block.

This design uses the alternate block, so the treatments used are $a, b, c, d, a b c, a b d, a c d, b c d$. We can split into blocks according to whether there is an even or odd number of As and Bs. Block 1: a, b, acd, bcd and Block 2: $c, d, a b c, a b d$.

