Confounding Two-Series Factorials

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Two-Series Factorials

Confounding is an incomplete blocking technique for factorial designs; we will discuss confounding for two-series designs.

Confounding in the two-series uses blocks of size 2^{k-j}.

For example, we could confound a 2^4 into two blocks of size 8 or four blocks of size 4 or eight blocks of size 2.

Like all incomplete blocking techniques, confounding has inefficiency.

Confounding's claim to fame is that it allows us to collect and isolate the inefficiency into one (or more) factorial effects and gives us 100% efficiency on the remaining factorial effects.

Colloquially, we throw some interactions under the bus in order to get good estimates on the other effects. Those interactions that went under the bus are not estimable at all; they're gone.

Specifically, those lost interactions are <u>confounded</u> with block differences.

Constructing the design

Begin with two blocks.

1. Choose an effect to be confounded. This is called the defining contrast.
2. Determine the $+/-$ pattern for the defining contrast.
3. Put the factor/level combinations that are $+$ on the defining contrast in one block and put the others (the -'s) in the second block.

To confound a 2^4 into two blocks you would typically choose to confound ABCD with blocks.

Block 1 (the $+$'s): (1), ab, ac, bc, ad, bd, cd, abcd
Block 2 (the –'s): a, b, c, abc, d, abd, acd, bcd

If instead you wanted to confound on ABC, then the blocks would be:

Block 1 (the $+$'s): a, b, c, abc, ad, bd, cd, abcd
Block 2 (the –'s): (1), ab, ac, bc, d, abd, acd, bcd

Alternatively, you can use an even/odd rule: does the treatment of interest contain an even or odd number of letters from the defining contrast.

To confound a 2^4 into two blocks confounding on ABCD:

Block 1 (the evens): (1), ab, ac, bc, ad, bd, cd, abcd
Block 2 (the odds): a, b, c, abc, d, abd, acd, bcd

If instead you wanted to block on ABC, then the blocks would be:

Block 1 (the odds): a, b, c, abc, ad, bd, cd, abcd
Block 2 (the evens): (1), ab, ac, bc, d, abd, acd, bcd

If you are a more binary kind of person, you can use the 011 sort of notation. Let x_A or x_B be 0 or 1 depending on whether A or B are high or low.

To confound on ABCD, use:
$x_A + x_B + x_C + x_D = 0 \mod 2$: (1), ab, ac, bc, ad, bd, cd, abcd
$x_A + x_B + x_C + x_D = 1 \mod 2$: a, b, c, abc, d, abd, acd, bcd

If instead you wanted to block on ABC, then the blocks would be:

$x_A + x_B + x_C = 1 \mod 2$: a, b, c, abc, ad, bd, cd, abcd
$x_A + x_B + x_C = 0 \mod 2$: (1), ab, ac, bc, d, abd, acd, bcd

The block with (1) or 0000 is called the principal block, and the other block is the alternate block. The labeling as block 1 or 2 is arbitrary.

I think that the even/odd method is easiest by hand. The modular arithmetic method is easiest to program on a computer.

Analysis is just like for a 2^k, except you must remember that one of the effects is block rather than treatment effect.

If you want 2^q blocks of size 2^{k-q}, then you need q defining contrasts (and not just any set of q, as we'll see below).

For example, consider using the even odd method with two defining contrasts to get four blocks.

For each factor level combination, determine whether it is even or odd on the two defining contrasts.

Split the treatments into four groups as: even/even, even/odd, odd/even, odd/odd.

Confound a 2^4 based on ACD and ABD as defining contrasts:

Trt	ACD	ABD
(1)	e	e
a	o	o
b	e	o
ab	o	e
c	o	e
ac	e	o
bc	o	o
abc	e	e
d	o	o
ad	e	e
bd	o	e
abd	e	o
cd	e	o
acd	o	e
bcd	e	e
abcd	o	o

e/e	e/o
(1)	b
abc	ac
ad	abd
bcd	cd

o/e	o/o
ab	a
c	bc
bd	d
acd	abcd

There are four blocks and thus three degrees of freedom between blocks.

ACD and ABD (the defining contrasts) are confounded; what else is confounded?

The generalized interaction is confounded. "Multiply" and then reduce exponents modulo 2.

$$ACD \times ABD = A^2B^1C^2D^2 = A^0B^1C^1D^0 = BC$$

BC is the third effect confounded with blocks.

Confounding generalized interactions is why we can't just choose the highest order effects as defining contrasts.

What if we had chosen ABCD and BCD to be defining contrasts? Then A would have been confounded, definitely not what we want.

For q defining contrasts, the confounded effects are the defining contrasts, all their two way generalized interactions, three way generalized interactions, up to q-way generalized interactions. That will be $2^q - 1$ effects in total.

Example: a 2^8 in 16 blocks of size 16. 15 confounded effects

Generators	2-way	3-way	4-way
ABCE	CDEF	AEFG	ABCEDFGH
ABDF	BDEG	BEFH	
ACDG	ADEH	CEGH	
BCDH	BCFG	DFGH	
	ACFH		
	ABGH		

Replication

There are two basic choices when replicating a confounded 2^k:

- Complete confounding means that you confound the same effects in every replication.
- Partial confounding means that you confound different effects in every replication.

The efficiency of the estimate is the fraction of replicates where the effect is not confounded. E.g., three replications and only confounded in one is $2/3$ efficiency. With partial confounding you can get non-zero efficiency for more effects. Complete confounding gives you zero efficiency for the completely confounded effects.

Analysis

For single replications, do analysis like any single replicate two series but recall that one or more of the effects are actually blocks.

For replicated confounding designs (partial or complete), simply look at treatments adjusted for blocks in the usual way.