Comment: "Unbiased Markov chain Monte Carlo with couplings by Pierre E. Jacob, John O'Leary, Yves F. Atchadé"

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We congratulate Jacob, O'Leary, and Atchadé on considerably furthering the cause of unbiased estimation in Markov chain Monte Carlo. Although there is often a need for the methods discussed in the paper, unbiased estimation is trivially achieved by successfully starting from stationarity. The authors mention coupling based perfect simulation techniques as a means to draw exact samples from the target distribution, but we note that simple accept-reject samplers are useful in surprisingly many examples. Accept-reject samplers can be inefficient for high-dimensional target distributions, but even there, in conjunction with *linchpin* variables (Archila, 2016; Huber, 2016), reasonably efficient accept-reject samplers can be used to produce one exact draw from the target.

For example, consider the model in Section 5.2 used to study the nuclear pump failure data. The joint posterior density of $\lambda = (\lambda_1, \ldots, \lambda_K)$ and β can factorized as

$$f(\lambda, \beta) = f(\lambda|\beta)\, f(\beta)\,.$$

Since it is easy to draw samples from $f(\lambda|\beta)$, β is a linchpin variable. Accept-reject samples from the univariate marginal posterior of β are easier to obtain than accept-reject samples from the $K+1$ dimensional joint posterior of (λ, β). In fact, a simple and efficient accept-reject sampler that targets $f(\beta)$ is easy to construct. Samples from the joint posterior can then be obtained via sequential sampling.

Based on a similar linchpin construction, Jones et al. (2006) presented an efficient accept-reject sampler for the baseball batting averages data and model presented in Section S3. If the number of covariates is small, efficient accept-reject samplers are also possible for the Bayesian lasso posterior in Section S5 and the variable selection model in Section 5.2. Archila (2016) present a few other hierarchical models where efficient accept-reject samplers can be implemented via the linchpin variable approach.

In many situations, accept-reject sampling is too computationally burdensome for a full Monte Carlo procedure, but it may be able to provide *one* draw at a reasonable cost. When this draw is used as a starting value in a Markov chain, the usual Monte Carlo estimator of *any* expectation is trivially unbiased. This procedure is easily parallelizable and requires no burn-in. However, in settings where direct sampling is inefficient, the unbiased estimation techniques of Jacob, O'Leary, and Atchadé are promising.

References

Archila, F. H. A. (2016). *Markov Chain Monte Carlo for Linear Mixed Models*. PhD thesis, University of Minnesota.

Huber, M. L. (2016). *Perfect simulation*. Chapman and Hall/CRC.

Jones, G. L., Haran, M., Caffo, B. S., and Neath, R. (2006). Fixed-width output analysis for Markov chain Monte Carlo. *Journal of the American Statistical Association*, 101:1537–1547.