Section 2.4 - 2.5 Probability (p.55)

2.54 Suppose that in a senior college class of 500 students it is found that 210 smoke, 258 drink alcoholic beverage, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97 smoke and eat between meals, and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student

(a) smokes but does not drink alcoholic beverages;

sol) Let $A$ be the event that students smoke, $B$ be the event that students drink alcoholic beverage, and $C$ be the event that students eat between meals.

\[
P(A \cap B') = P(A) - P(A \cap B) = \frac{210}{500} - \frac{122}{500} = \frac{88}{500}
\]

(b) eats between meals and drinks alcoholic beverages but does not smoke;

sol) \[
P(C \cap B \cap A') = P(B \cap C) - P(A \cap B \cap C) = \frac{83}{500} - \frac{52}{500} = \frac{31}{500}
\]

(c) neither smokes nor eats between meals.

sol) \[
P((A \cup C)') = 1 - P(A \cup C) = 1 - \frac{329}{500} = \frac{171}{500}
\]

2.56 From past experiences a stockbroker believes that under present economic conditions a customer will invest in tax-free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax-free bonds and mutual funds with a probability of 0.15. At this time, find the probability that a customer will invest

(a) in either tax-free bond or mutual funds;

sol) Let $A$ be an event that a customer will invest in tax-free bonds and $B$ be an event that a customer will invest in mutual funds.

Then, $P(A) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.15$.

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.3 - 0.15 = 0.75
\]

(b) in neither tax-free bonds nor mutual funds.

sol) \[
P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.75 = 0.25
\]
2.60 A pair of fair dice is tossed. Find the probability of getting

(a) a total of 8;

sol) Let $A$ be an event that getting a total of 8.
$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$
$P(A) = \frac{5}{36}$

(b) at most a total of 5.

sol) Let $B$ be an event that getting at most of a total of 5.
$B = \{(1,1), (1,2), (2,1), (1,3), (2,2), (3,1), (1,4), (2,3), (3,2), (4,1)\}$
$P(B) = \frac{10}{36} = \frac{5}{18}$

2.62 If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems, and a dictionary, what is the probability that

(a) the dictionary is selected?

sol) Let $A$ be an event that the dictionary is selected.
$n = \binom{1}{1}\binom{8}{2} = \frac{8!}{2!6!} = 28$
$N = \binom{9}{3} = \frac{9!}{3!6!} = 84$
$P(A) = \frac{n}{N} = \frac{28}{84} = \frac{1}{3}$

(b) 2 novels and 1 book of poems are selected?

sol) Let $B$ be an event that 2 novels and 1 book of poems are selected.
$n = \binom{5}{2}\binom{3}{1} = \frac{5!}{2!3!} \times \frac{3!}{1!2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1 \times 1} \times \frac{3 \times 2 \times 1}{2 \times 1} = 30$
$N = \binom{9}{3} = \frac{9!}{3!6!} = 84$
$P(B) = \frac{n}{N} = \frac{30}{84} = \frac{5}{14}$

2.63 In a poker band consisting of 5 cards, find the probability of holding

(a) 3 aces;

sol) $n = \binom{4}{3}\binom{48}{2} = \frac{4!}{3!1!} \times \frac{48!}{2!46!} = \frac{4 \times 3 \times 2 \times 1 \times 48 \times 47}{2 \times 1 \times 2 \times 1} = 4512$
$N = \binom{52}{5} = \frac{52!}{5!47!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2598960$
$P(A) = \frac{n}{N} = \frac{4512}{2598960} = \frac{94}{54035} = 0.001736$

(b) 4 hearts and 1 club.
2.64 In a game of Yahtzee, where 5 dice are tossed simultaneously, find the probability of getting 4 of a kind.

sol)
Any four of a kind, say four 2’s and one 5 occur in \( \binom{5}{1} \) = 5 ways each with probability \( \left( \frac{1}{6} \right)^4 \left( \frac{1}{6} \right) = \left( \frac{1}{6} \right)^5 \). Since there are \( 6 \times 5 = 30 \) ways to choose various pairs of numbers to constitute four of one kind and one of the other (we use permutation instead of combination is because that four 2’s and one 5, and four 5’s and one 2 are two different ways), the probability is \( \frac{5}{30} \times \frac{1}{6} = \frac{25}{1296} \).

Section 2.6 - 2.7 Conditional probability (p.65)

2.79 A random sample of 200 adults are classified below by sex and their level of education attained.
If a person is picked at random from this group, find the probability that
(a) the person is a male, given that the person has a secondary education;

sol)
Let \( M \) be an event that the person is a male and \( S \) be an event that the person has a secondary education.
\[
P(S) = \frac{28 + 50}{200} = \frac{78}{200}
\]
\[
P(M \cap S) = \frac{28}{200}
\]
\[
P(M | S) = \frac{P(M \cap S)}{P(S)} = \frac{28}{78} = \frac{14}{39}
\]

(b) the person does not have a college degree, given that the person is a female.

sol)
Let \( F \) be an event that the person is a female and \( C \) be an event that the person have a college degree.
\[
P(F) = \frac{45 + 50 + 17}{200} = \frac{112}{200}
\]
\[
P(C' \cap F) = \frac{35 + 50}{200} = \frac{85}{200}
\]
\[
P(C' | F) = \frac{P(C' \cap F)}{P(F)} = \frac{85}{112} = \frac{95}{112}
\]
2.85 The probability that a married man watches a certain television show is 0.4 and the probability that a married woman watches the show is 0.5. The probability that a man watches the show, given that his wife does, is 0.7. Find the probability that

(a) a married couple watches the show;

\[
P(M \cap W) = P(M)P(M \mid W) = 0.4 \times 0.7 = 0.28
\]

(b) a wife watches the show given that her husband does;

\[
P(W \mid M) = \frac{P(M \cap W)}{P(M)} = \frac{0.28}{0.4} = 0.7
\]

(c) at least 1 person of a married couple will watch the show.

\[
P(W \cup M) = P(M) + P(W) - P(M \cap W) = 0.4 + 0.5 - 0.28 = 0.62
\]

2.93 A town has 2 fire engines operating independently. The probability that a specific engine is available when needed is 0.96.

(a) What is the probability that neither is available when needed?

\[
P(A'_1 \cap A'_2) = P(A'_1)P(A'_2) = (1 - P(A_1))(1 - P(A_2)) = (1 - 0.96)(1 - 0.96) = 0.04 \times 0.04 = 0.0016
\]

(b) What is the probability that a fire engine is available when needed?

\[
P(A_1 \cup A_2) = 1 - P(A'_1 \cap A'_2) = 1 - 0.0016 = 0.9984
\]

2.97 Find the probability of randomly selecting 4 good quarts of milk in succession from a cooler containing 20 quarts of which 5 have spoiled, by using

(a) the first formula of Theorem 2.15 on page 64:

\[
P(\text{4 good quarts}) = \frac{15}{20} \times \frac{14}{19} \times \frac{13}{18} \times \frac{12}{17} = \frac{3,080}{20,940} = 0.1476
\]
Let $A_1$ be an event that the first quart of milk is good, $A_2$ be an event that the second quart of milk is good, $A_3$ be an event that the third quart of milk is good, and $A_4$ be an event that the forth quart of milk is good.

\[ P(A_1) = \frac{15}{20}, \quad P(A_2 \mid A_1) = \frac{14}{19}, \quad P(A_3 \mid A_1 \cap A_2) = \frac{13}{18}, \quad P(A_4 \mid A_1 \cap A_2 \cap A_3) = \frac{12}{17} \]

\[ P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2)P(A_4 \mid A_1 \cap A_2 \cap A_3) = \frac{15}{20} \times \frac{14}{19} \times \frac{13}{18} \times \frac{12}{17} = \frac{91}{323} \]

(b) the formulas of Theorem 2.8 and 2.9 on pages 46 and 50, respectively.

Let $A$ be an event that the 4 quarts of milk are good.

\[ n = \binom{15}{4} = \frac{15!}{4!11!} \]

\[ N = \binom{20}{4} = \frac{20!}{4!16!} \]

\[ P(A) = \frac{n}{N} = \frac{15! \times 14 \times 13 \times 12}{20! \times 19 \times 18 \times 17} = \frac{91}{323} \]

\[ P(C) = 0.05, P(D \mid C) = 0.78, P(D \mid C') = 0.06 \]

\[ P(D) = P(D \cap C) + P(D \cap C') = P(C)P(D \mid C) + P(C')P(D \mid C') = 0.05 \times 0.78 + 0.95 \times 0.06 = 0.0390 + 0.0570 = 0.0960 \]

2.101 In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that a person is diagnosed as having cancer?

\[ P(C \mid D) = \frac{P(D \cap C)}{P(D)} = \frac{P(C)P(D \mid C)}{0.05 \times 0.78} = \frac{0.05 \times 0.78}{0.0960} = \frac{0.0390}{0.0960} = 0.40625 \]

2.103 Referring to Exercise 2.101, what is the probability that a person diagnosed as having cancer actually has the disease?

\[ P(C \mid D) = \frac{P(D \cap C)}{P(D)} = \frac{P(C)P(D \mid C)}{P(D)} = \frac{0.05 \times 0.78}{0.0960} = \frac{0.0390}{0.0960} = 0.40625 \]

2.107 Pollution of the rivers in the United States has been a problem for many years. Consider the following events:
A = The river is polluted.
B = A sample of water tested detects pollution.
C = Fishing permitted.

Assume $P(A) = 0.3$, $P(B | A) = 0.75$, $P(B | A') = 0.20$, $P(C | A \cap B) = 0.20$, $P(C | A' \cap B) = 0.15$, $P(C | A \cap B') = 0.80$, and $P(C | A' \cap B') = 0.90$.

(a) Find $P(A \cap B \cap C)$.

sol)
$P(A) = 0.3$
$P(A \cap B) = P(A)P(B | A) = 0.3 \times 0.75 = 0.225$
$P(A \cap B \cap C) = P(A \cap B)P(C | A \cap B) = 0.225 \times 0.20 = 0.045$

(b) Find $P(B' \cap C)$.

sol)
$P(A' \cap B) = P(A')P(B | A') = (1 - 0.3) \times 0.20 = 0.14$
$P(A' \cap B \cap C) = P(A' \cap B)P(C | A' \cap B) = 0.14 \times 0.15 = 0.021$
$P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.225 = 0.075$
$P(A \cap B' \cap C) = P(A \cap B')P(C | A \cap B') = 0.075 \times 0.80 = 0.06$
$P(B') = P(A \cup B) + P(A' \cap B) = 0.225 + 0.14 = 0.365$
$P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C) = 0.06 + 0.504 = 0.564$

(c) Find $P(C)$.

sol)
$P(C) = P(B' \cap C) + P(A \cap B \cap C) + P(A' \cap B \cap C) = 0.564 + 0.045 + 0.021 = 0.630$

(d) Find the probability that the river is polluted, given that fishing is permitted and the sample tested did not detect pollution.

sol)
$P(A | C \cap B') = \frac{P(A \cap B' \cap C)}{P(B' \cap C)} = \frac{0.06}{0.564} = 0.1064$