Stat 5101 First Midterm Exam **February 27, 2019**

Name ______________________ Student ID ______________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. You may use a calculator. No other electronic devices are allowed.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from. Simplify formulas as much as you easily can, but there is no unique "correct" simplification. Any correct answer gets full credit unless the question explicitly states otherwise.

Abbreviations used: probability mass function (PMF).

The points for the questions total to 100. There are 5 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having PMF with parameter $\theta \ge 0$ given by

x	-2	-1	0	1	2
$f_\theta(x)$	$\frac{\theta^2}{1+2\theta+2\theta^2}$	$\frac{\theta}{1+2\theta+2\theta^2}$	$\frac{1}{1+2\theta+2\theta^2}$	$\frac{\theta}{1+2\theta+2\theta^2}$	$\frac{\theta^2}{1+2\theta+2\theta^2}$

In this problem simplify your answers so they do not leave undone a sum over the points in the sample space.

(a) Calculate $E_\theta(X)$.

(b) Calculate $\operatorname{var}_\theta(X)$.

2. [20 pts.] Suppose X is a random variable having PMF with parameter $\theta \geq 0$ given by

x	0	1	2	3	4
$f_\theta(x)$	$\frac{1-\theta}{1-\theta^5}$	$\frac{(1-\theta)\theta}{1-\theta^5}$	$\frac{(1-\theta)\theta^2}{1-\theta^5}$	$\frac{(1-\theta)\theta^3}{1-\theta^5}$	$\frac{(1-\theta)\theta^4}{1-\theta^5}$

Find the PMF of the random variable $Y = X(X-2)(X-4)$.

3. [20 pts.] Suppose $\mathbf{X}$ is a random vector with mean vector

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_1 \\ \mu_2 \\ \mu_2 \end{pmatrix}$$

and variance matrix

$$\mathbf{M} = \begin{pmatrix} \sigma_1^2 & \rho_1 & 0 & 0 \\ \rho_1 & \sigma_1^2 & 0 & 0 \\ 0 & 0 & \sigma_2^2 & \rho_2 \\ 0 & 0 & \rho_2 & \sigma_2^2 \end{pmatrix}$$

where μ_1, μ_2, σ_1, σ_2, ρ_1, and ρ_2 are real numbers satisfying $|\rho_1| \le \sigma_1^2$ and $|\rho_2| \le \sigma_2^2$.

(a) Find $E(X_1 + X_2 + X_3 + X_4)$.

(b) Find $\operatorname{var}(X_1 + X_2 + X_3 + X_4)$.

4. [20 pts.] Suppose X and Y are independent and uniformly distributed on the set $S = \{1, 2, 3\}$. In each part give reasoning justifying your answer.

   (a) Are X^2 and Y^3 independent?

   (b) Are X and $X + Y$ independent?

   (c) Are $X - Y$ and $X + Y$ independent?

5. [20 pts.] In this problem, simplify your answers so they do not contain any unevaluated binomial coefficients.

(a) There are 3 red and 1 white balls in an urn, and we draw a random sample of size 2 with replacement from the urn (this means the balls are well mixed before each draw). If X is the number of red balls drawn, what is the PMF of X?

(b) Exactly the same question as in part (a) except that we change with replacement to without replacement (in which case it does not matter whether the balls are well mixed between draws as long as they were well mixed before the first draw).