Stat 5101 Second Midterm Exam **April 6, 2016**

Name ______________________ Student ID ______________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers or formulas with no indication of where they came from. Leave no undone integrals in your answers, but other than that requirement there is no unique "correct" simplification. Any correct answer gets full credit unless the question explicitly states otherwise.

Abbreviations used: probability density function (PDF); probability mass function (PMF).

The points for the questions total to 100. There are 5 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having PDF given by

$$f_\theta(x) = \frac{60x^2(1-x)(\theta+x)}{3+5\theta}, \qquad 0 < x < 1,$$

where $\theta \geq 0$ is a parameter.

(a) Find the mean of X.

(b) Find the probability of the event $X < 1/2$.

2. [20 pts.] Suppose X is a random variable having a Beta(α_1, α_2) distribution. Find the PDF of the random variable $Y = 1/(1-X)$. The definition of a function describes the domain as well as the rule.

3. [20 pts.] Suppose X is a random variable having PDF given by

$$f_\alpha(x) = \alpha x^{\alpha-1}, \qquad 0 < x < 1,$$

where $\alpha > 0$ is a parameter.

(a) Find its distribution function (DF). Be sure to define the DF on the whole real line.

(b) Find the median of the distribution of X.

4. [20 pts.] Suppose the random vector (X, Y) has the PDF

$$f_\theta(x, y) = \frac{6\theta}{(1 + \theta x + y)^4}, \qquad 0 < x < \infty,\ 0 < y < \infty,$$

where $\theta > 0$ is a parameter.

(a) Find the conditional PDF of Y given X.

(b) Find the conditional expectation of Y given X. (Hint: do the integral either by substitution or by integration by parts).

5. [20 pts.] Suppose the conditional PMF of Y given X is

$$f(y \mid x) = x(1-x)^{y-1}, \qquad y = 1, 2, \ldots.$$

and suppose the marginal distribution of X is $\text{Beta}(\alpha_1, \alpha_2)$ where $\alpha_1 > 0$ and $\alpha_2 > 0$. What is the conditional distribution of X given Y? Since this is a brand name distribution, no integrals need be done, it is enough to name the distribution and give its parameters as a function of y and α_1 and α_2.