Stat 5101 First Midterm Exam **February 24, 2016**

Name ______________________________ Student ID ____________________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from. Simplify formulas as much as you easily can, but there is no unique "correct" simplification. Any correct answer gets full credit unless the question explicitly states otherwise.

Abbreviations used: probability mass function (PMF).

The points for the questions total to 100. There are 7 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having PMF with parameter $\theta \geq 0$ given by

x	1	2	3	4	5
$f_\theta(x)$	$\frac{1+\theta}{9+5\theta}$	$\frac{2+\theta}{9+5\theta}$	$\frac{3+\theta}{9+5\theta}$	$\frac{2+\theta}{9+5\theta}$	$\frac{1+\theta}{9+5\theta}$

In this problem simplify your answers so they do not leave undone a sum over the points in the sample space.

(a) Calculate $E_\theta(X)$.

(b) Calculate $\text{var}_\theta(X)$.

2. [20 pts.] Suppose X is a random variable having PMF with parameter $\theta \geq 0$ given by

x	1	2	3	4	5
$f_\theta(x)$	$\frac{1-\theta}{1-\theta^5}$	$\frac{\theta(1-\theta)}{1-\theta^5}$	$\frac{\theta^2(1-\theta)}{1-\theta^5}$	$\frac{\theta^3(1-\theta)}{1-\theta^5}$	$\frac{\theta^4(1-\theta)}{1-\theta^5}$

Find the PMF of the random variable $Y = |X - 3|$.

3. [20 pts.] Suppose $\mathbf{X}$ is a random vector with mean vector

$$\boldsymbol{\mu} = \begin{pmatrix} \mu \\ \mu \\ \mu \\ \mu \end{pmatrix}$$

and variance matrix

$$\mathbf{M} = \begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{pmatrix}$$

(a) Find $E(X_1 + X_2 + X_3 + X_4)$.

(b) Find $\operatorname{var}(X_1 + X_2 + X_3 + X_4)$.

4. [20 pts.] Suppose the random vector (X, Y, Z) has PMF given by each of the following definitions. In all parts of this question $S = \{-2, -1, 0, 1, 2\}$. In each part say whether X, Y, and Z are independent random variables, and explain why or why not, as the case may be. In each part say clearly what the support of the random vector (X, Y, Z) is.

(a)

$$f(x, y, z) = \frac{4 - x + y}{500}, \qquad (x, y, z) \in S^3$$

(b)

$$f(x, y, z) = \frac{4 - x + y}{175}, \qquad (x, y, z) \in S^3 \text{ and } x \leq y \leq z$$

(c)

$$f(x, y, z) = \frac{x^2 z^2}{500}, \qquad (x, y, z) \in S^3$$

(d)

$$f(x, y, z) = \frac{x^2 z^2}{165}, \qquad (x, y, z) \in S^3 \text{ and } x \leq y \leq z$$

(e)

$$f(x, y, z) = c_1 \exp(x + y + z), \qquad (x, y, z) \in S^3$$

where exp denotes the exponential function ($\exp(w) = e^w$) and where c_1 is the constant that makes the PMF sum to one ($c_1 \approx 6.39 \times 10^{-4}$)

(f)

$$f(x, y, z) = c_2 \exp(x + y + z), \qquad (x, y, z) \in S^3 \text{ and } x \leq y \leq z$$

similarly for c_2 ($c_2 \approx 1.30 \times 10^{-3}$)

5. [20 pts.] In this problem, simply your answers so they do not contain any unevaluated binomial coefficients.

   (a) There are 3 red and 2 white balls in an urn, and we draw a random sample of size 3 with replacement from the urn (this means the balls are well mixed before each draw). What is the probability of obtaining either zero or one red balls?

   (b) Exactly the same question as in part (a) except that we change with replacement to without replacement (in which case it does not matter whether the balls are well mixed between draws as long as they were well mixed before the first draw).