Stat 5102 First Midterm Exam　　　　　　　　**February 25, 2015**

Name ――――――――――――――――― Student ID ――――――――――

　　The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. You may use a calculator.

　　Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for answers with no indication of where they came from. Leave no undone integrals in your answers, but other than that requirement there is no unique "correct" simplification. Any correct answer gets full credit, except as explicitly stated in questions.

　　Abbreviations used: asymptotic relative efficiency (ARE), distribution function (DF), probability density function (PDF), independent and identically distributed (IID).

　　The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] For the following data

$$-12.5 \quad -6.5 \quad -2.0 \quad -1.0 \quad 2.0$$

(a) Find the mean of the empirical distribution.

(b) Find the variance of the empirical distribution.

(c) Find a median of the empirical distribution.

(d) Find a lower quartile of the empirical distribution.

(e) Find the probability that X is strictly greater than zero when X is a random variable having the empirical distribution.

2. [20 pts.] The function

$$F_\theta(x) = \begin{cases} 0, & x \le 0 \\ \frac{2}{1+\exp(-\theta x)} - 1, & 0 < x < \infty \end{cases}$$

is a DF (not a PDF), where the parameter θ is any positive real number. Find the asymptotic distribution of the sample median of an IID sample having this distribution.

3. [20 pts.] Suppose X_1, X_2, ... are IID Geo(p) random variables, and suppose we are interested in estimating the parameter

$$\theta = \log(1 - p).$$

(a) Find a method of moments estimator of θ.

(b) Find the asymptotic normal distribution of your method of moments estimator.

(c) Express the asymptotic variance in terms of the parameter θ only (no other parameters, no p, μ, or σ^2).

4. [20 pts.] The function

$$f_\theta(x) = \frac{\sqrt{2}}{\pi[1 + (x - \theta)^4]}, \qquad -\infty < x < \infty$$

is a PDF when the parameter θ is any real number. The variance of this distribution is

$$\text{var}_\theta(X) = 1$$

(you do not have to prove this).

(a) Show that this distribution is symmetric and θ is the center of symmetry.

(b) Find the ARE of the sample mean and sample median of an IID sample from this distribution, both considered as estimators of the center of symmetry. Also state which estimator is better.

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5. [20 pts.] Suppose X_1, X_2, ... are IID from the $\mathrm{Gam}(4, \lambda)$ distribution. Find an asymptotic 95% confidence interval for λ, the endpoints of which are a function of $\overline{X}_n$ only (no other statistics). Hint: the 0.975 quantile of the standard normal distribution is 1.9600.