Stat 5102 First Midterm Exam **February 27, 2013**

Name ______________________________ Student ID __________________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on “brand name distributions” and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] For the following data

$$1.5 \quad 2.0 \quad 2.5 \quad 3.0 \quad 4.5 \quad 6.0 \quad 12.0$$

(a) Find the mean of the empirical distribution.

(b) Find the variance of the empirical distribution.

(c) Find a median of the empirical distribution.

(d) Find a lower quartile of the empirical distribution.

(e) Find the probability that X is greater ($>$ not $\geq$) than its mean when X is a random variable having the empirical distribution.

2. [20 pts.] The function

$$f_\theta(x) = \theta x^{\theta - 1}, \qquad 0 < x < 1$$

is a probability density function (PDF) (note not a distribution function (DF)) where the parameter θ is any positive real number. Find the asymptotic distribution of the sample median of an independent and identically distributed (IID) sample having this distribution.

3. [20 pts.] Suppose X_1, X_2, ... are IID Ber(p) random variables, and suppose we are interested in estimating the parameter

$$\theta = \log(p) - \log(1 - p).$$

(a) Find a method of moments estimator of θ.

(b) Find the asymptotic normal distribution of your method of moments estimator.

(c) Express the asymptotic variance in terms of θ rather than p.

4. [20 pts.] The function

$$f_{\mu,\sigma}(x) = \frac{1}{\sigma} \cdot \frac{e^{(x-\mu)/\sigma}}{(1 + e^{(x-\mu)/\sigma})^2}, \qquad -\infty < x < \infty$$

is a PDF when the parameter θ is any positive real number. The variance of this distribution is

$$\operatorname{var}_\theta(X) = \frac{\pi^2 \sigma^2}{3}$$

(you do not have to prove this).

(a) Show that this distribution is symmetric and μ is the center of symmetry.

(b) Find the asymptotic relative efficiency (ARE) of the sample mean and sample median of an IID sample from this distribution, both considered as estimators of the center of symmetry. Also state which estimator is better.

5. [20 pts.] Suppose X_1, X_2, $\ldots$ are IID from the $\text{Poi}(\mu)$ distribution, and suppose we are interested in estimating the parameter

$$\theta = \log(\mu).$$

Find an asymptotic 95% confidence interval for θ, the endpoints of which are a function of $\overline{X}_n$ only (no other statistics). Hint: the 0.975 quantile of the standard normal distribution is 1.9600.