Stat 5102 Second Midterm Exam **April 4, 2012**

Name ______________________________ Student ID __________________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

IID means independent and identically distributed; PDF means probability density function; MLE means maximum likelihood estimate.

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] Suppose $X_1, \ldots, X_n$ are IID from the distribution having PDF

$$f_\theta(x) = \frac{\theta}{(1 + x)^{\theta+1}}, \qquad x > 0,$$

where θ is an unknown parameter satisfying $\theta > 0$.

(a) Find the log likelihood for θ.

(b) Find the MLE for θ.

(c) Show that your MLE is the unique global maximizer of the log likelihood.

2. [20 pts.] Suppose X_1, …, X_n are IID NegBin(r, p), where r is a known positive integer and p is the unknown parameter.

   (a) Find the log likelihood for p.

   (b) Find the MLE for p.

   (c) Find the asymptotic distribution of your MLE. The mean and variance of the asymptotic normal distribution should be expressed as functions of p, r, and n only.

3. [20 pts.] Suppose $X_1, \ldots, X_n$ are IID with PDF

$$f(x \mid \theta) = \theta x^{\theta - 1}, \qquad 0 < x < 1,$$

where $\theta > 0$ is an unknown parameter. Suppose the prior distribution for θ is $\text{Gam}(\alpha_0, \lambda_0)$, where α_0 and λ_0 are hyperparameters. Find the posterior distribution for θ. (Hint: laws of exponents and logarithms.)

4. [20 pts.] Suppose $X_1, \ldots, X_n$ are IID $\mathcal{N}(\mu, 1/\lambda)$, where μ is a known number and $\lambda > 0$ is the unknown parameter. Suppose we use the improper distribution with density function

$$g(\lambda) = \frac{1}{\lambda}$$

(a) Find the posterior distribution for λ.

(b) For what values of the data x does your answer to part (a) make sense?

5. [20 pts.] Show that the Beta(α_1, α_2) family of distributions is an exponential family. Identify the natural parameter vector and the natural statistic vector. (Hint: since this is a two-parameter family, both will be two-dimensional.)