

Name \_\_\_\_\_ Student ID \_\_\_\_\_

The exam is closed book and closed notes. You may use one  $8\frac{1}{2} \times 11$  sheet of paper with formulas, etc. You may also use the handouts on “brand name distributions” and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] For the following data

1.5 2.0 2.5 3.0 6.0

- (a) Find the mean of the empirical distribution.

- (b) Find the standard deviation of the empirical distribution.

(c) Find a median of the empirical distribution.

(d) Find a lower quartile of the empirical distribution.

(e) Find an upper quartile of the empirical distribution.

2. [20 pts.] The function

$$F_{\theta}(x) = \begin{cases} 0, & x \leq 0 \\ x/(\theta + x), & 0 < x \end{cases}$$

is a DF (note not PDF) when the parameter  $\theta$  is any positive real number. Find the asymptotic distribution of the sample median of an independent and identically distributed (IID) sample having this distribution.

3. [20 pts.] The function

$$f_{\theta}(x) = \frac{2x + 2\theta}{1 + 2\theta}, \quad 0 < x < 1,$$

is a PDF when the parameter  $\theta$  is any positive real number. The mean and variance of this distribution are

$$E_{\theta}(X) = \frac{2 + 3\theta}{3 + 6\theta}$$
$$\text{var}_{\theta}(X) = \frac{1}{12} - \frac{1}{36(1 + 2\theta)^2}$$

(you do not have to prove these). Suppose  $X_1, X_2, \dots$  are IID from this distribution.

(a) Find a method of moments estimator of  $\theta$ .

(b) Find the asymptotic normal distribution of your method of moments estimator.

4. [20 pts.] The  $\text{UniformDis}(0, \theta)$  distribution is symmetric with center of symmetry  $\theta/2$ . Find the asymptotic relative efficiency (ARE) of the sample mean and sample median of an IID sample from this distribution, both considered as estimators of  $\theta/2$ . Also state which is the estimator is better.

5. [20 pts.] Suppose  $X_1, X_2, \dots$  are IID from the  $\text{Beta}(\theta, 4 - \theta)$  distribution, where  $\theta$  is an unknown parameter satisfying  $0 < \theta < 4$ . Since  $E(X_i) = \theta/4$ , the obvious method of moments estimator of  $\theta$  is  $4\bar{X}_n$ . Find an asymptotic 95% confidence interval for  $\theta$ , the endpoints of which are a function of  $\bar{X}_n$  only (no other statistics). Hint: the 0.975 quantile of the standard normal distribution is 1.9600.