

Stat 5102 (Geyer) Spring 2012
Homework Assignment 1
Due Wednesday, January 25, 2012

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

1-1. For the following data

2.2 0.8 2.0 3.6 3.6 1.8 2.2 1.0 1.2 2.4
1.4 3.2 1.0 1.8 1.6 0.4 0.2 2.0 2.2 2.4

- (a) Find the mean of the empirical distribution.
- (b) Find the variance of the empirical distribution.
- (c) Find the standard deviation of the empirical distribution.
- (d) Find the median of the empirical distribution.
- (e) Find the lower and upper quartiles (the 0.25 and 0.75 quantiles) of the empirical distribution.
- (f) Plot the empirical distribution function.
- (g) Find $\Pr(X < 2.5)$ under the empirical distribution.

1-2. Suppose n is a positive integer, p is a real number such that $0 \leq p \leq 1$ and np is an integer. Suppose the data consist of $n(1-p)$ zeros and np ones. Show that

$$E_n(X) = p$$
$$\text{var}_n(X) = p(1-p)$$

Hint: no calculation necessary if you apply the theory you know from 5101.

1-3. The *median absolute deviation from the median* (MAD) of a random variable X with unique median m is the median of the random variable $Y = |X - m|$. The MAD of the values x_1, \dots, x_n is the median of the values $|x_i - \tilde{x}_n|$, where \tilde{x}_n is the empirical median defined on Slide 20, Deck 1. The *interquartile range* of a random variable X with unique quartiles is the difference upper quartile minus lower quartile.

- (a) Show that for a symmetric continuous random variable with strictly positive PDF the MAD is half the interquartile range. (The point of requiring a strictly positive PDF is that this makes all the quantiles unique and distinct.
- (b) Calculate the MAD for the standard normal distribution.
- (c) Calculate the MAD for the standard Cauchy distribution.
- (d) Calculate the MAD for the data in Problem 1-1.

1-4. Show that if X_1, \dots, X_n are IID $\text{Gam}(\alpha, \lambda)$, then

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

has the $\text{Gam}(n\alpha, n\lambda)$ distribution. Hint: use the addition rule for the gamma distribution and the change-of-variable formula for the change-of-variable $x \mapsto x/n$.

1-5. Show that if X has the $t(\nu)$ distribution, then X^2 has the $F(1, \nu)$ distribution.

1-6. Show that if X has the $F(\mu, \nu)$ distribution and $\nu > 2$, then

$$E(X) = \frac{\nu}{\nu - 2}$$

1-7. Show that if X has the $t(\nu)$ distribution and $\nu > 2$, then

$$\text{var}(X) = \frac{\nu}{\nu - 2}$$