Stat 5102 Second Midterm Exam **April 7, 2010**

Name ______________________ Student ID ______________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

IID means independent and identically distributed; PDF means probability density function; MLE means maximum likelihood estimate.

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] Suppose X_1, $\ldots$, X_n are IID $\mathcal{N}(0, \nu)$, where ν is an unknown parameter satisfying $\nu > 0$.

   (a) Find the log likelihood for ν.

   (b) Find the MLE for ν.

(c) Show that your MLE is a local maximizer of the log likelihood.

2. [20 pts.] Suppose $X_1, \ldots, X_n$ are IID having PDF

$$f_\theta(x) = \theta(\theta + 1)x^{\theta-1}(1 - x), \qquad 0 < x < 1$$

where $\theta > 0$ is an unknown parameter.

(a) Find the log likelihood for θ.

(b) Find the MLE for θ. **Hint:** the roots of the quadratic polynomial $ax^2 + bx + c$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(c) Find the asymptotic distribution of your MLE. The mean and variance of the asymptotic normal distribution should be expressed as functions of θ and n only.

3. [20 pts.] Suppose $X_1, \ldots, X_n$ are IID $\mathcal{N}(0, 1/\lambda)$, and suppose the prior distribution for λ is $\text{Gam}(\alpha_0, \lambda_0)$, where α_0 and λ_0 are hyperparameters. Find the posterior distribution for λ.

4. [20 pts.] Suppose X is $\mathcal{N}(\mu, 1)$. We have only one observation. And suppose the prior distribution for μ is proportional to $\exp(\mu)$, an improper prior.

   (a) Find the posterior distribution for μ.

   (b) For what values of the data x does your answer to part (a) make sense?

5. [20 pts.] Suppose X_1, …, X_n are IID Exp(λ), where λ is an unknown parameter. Describe how to do an exact (not approximate) test of the hypotheses

$$
\begin{aligned}
H_0 &: \lambda = \lambda_0 \\
H_1 &: \lambda < \lambda_0
\end{aligned}
$$

where λ_0 is a specified number (the value of λ hypothesized under H_0. Give a formula for the P-value of the test, an expression in terms of functions you can find in the R computer language (or Rweb). **Hint:** Consider the relationship between $E(X_i)$ and λ.