Stat 5102 First Midterm Exam **February 24, 2010**

Name ______________________________ Student ID ____________________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 5 pages and 5 problems.

1. [20 pts.] For the following data

$$1.5 \quad 2.0 \quad 2.5 \quad 3.0 \quad 4.5 \quad 7.0 \quad 11.5 \quad 16$$

(a) Find the mean of the empirical distribution.

(b) Find a median of the empirical distribution.

(c) Find a 0.25 quantile of the empirical distribution.

2. [20 pts.] The function

$$F_\theta(x) = \begin{cases} 0, & x \le 1 \\ 1 - x^{-\theta}, & x > 1 \end{cases}$$

is a distribution function (note: DF not PDF) when the parameter θ is any positive real number. Find the asymptotic distribution of the sample median of an independent and identically distributed (IID) sample having this distribution.

3. [20 pts.] The function

$$f_\theta(x) = \theta x^{-\theta - 1}, \qquad x > 1$$

is a PDF when the parameter θ is any positive real number. If $\theta > 2$, then the mean and variance of this distribution are

$$\begin{aligned} E_\theta(X) &= \frac{\theta}{\theta - 1} \\ \operatorname{var}_\theta(X) &= \frac{\theta}{(\theta - 2)(\theta - 1)^2} \end{aligned}$$

Suppose X_1, X_2, ... are IID from this distribution. The obvious method of moments estimator, derived from the first of these, is

$$\hat{\theta}_n = \frac{\overline{X}_n}{\overline{X}_n - 1}$$

(you do not have to prove these).

Find the asymptotic normal distribution of $\hat{\theta}_n$ assuming $\theta > 2$.

4. [20 pts.] The function

$$f_\theta(x) = \frac{3}{4}[1 - (x - \theta)^2], \qquad \theta - 1 < x < \theta + 1$$

is a PDF, where the parameter θ can be any real number. Find the asymptotic relative efficiency (ARE) of the sample mean and sample median of an IID sample from this distribution, both considered as estimators of θ. Also state which is the estimator is better.

5. [20 pts.] Suppose X_1, X_2, $\ldots$ are IID $\text{Gam}(\alpha, 1)$, where $\alpha > 0$ is an unknown parameter. Since $E(X) = \alpha$, the obvious method of moments estimator of α is $\overline{X}_n$. Find an asymptotic 95% confidence interval for α, the endpoints of which are a function of $\overline{X}_n$ only (no other statistics). Hint: the 0.975 quantile of the standard normal distribution is 1.9600.