

Stat 5102 (Geyer) Spring 2010  
Homework Assignment 3  
Due Wednesday, February 10, 2010

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

**3-1.** Show that the family of  $\text{Gam}(\alpha, \lambda)$  distributions with  $\alpha$  known and  $\lambda$  unknown, so the parameter space is

$$\{\lambda \in \mathbb{R} : \lambda > 0\}$$

is a scale family.

**3-2.** Suppose  $S_n^2$  is the sample variance calculated from an IID normal random sample of size  $n$ .

- (a) Calculate the bias of  $S_n$  as an estimator of the population standard deviation  $\sigma$ .
- (b) Find the constant  $a$  such that  $aS_n$  has the smallest mean square error as an estimator of  $\sigma$ .

**3-3.** Suppose  $U$  and  $V$  are statistics that are independent random variables and both are unbiased estimators of a parameter  $\theta$ . Write  $\text{var}(U) = \sigma_U^2$  and  $\text{var}(V) = \sigma_V^2$ , and define another statistic  $T = aU + (1 - a)V$  where  $a$  is an arbitrary but known constant.

- (a) Show that  $T$  is an unbiased estimator of  $\theta$ .
- (b) Find the  $a$  that gives  $T$  the smallest mean square error.

**3-4.** The slides don't give any examples of estimators that are *not* consistent. Give an example of an inconsistent estimator of the population mean.

**3-5.** If  $X \sim \text{Bin}(n, p)$ , show that  $\hat{p}_n = X/n$  is a consistent and asymptotically normal estimator of  $p$ , and give the asymptotic distribution of  $\hat{p}_n$ .

**3-6.** If  $X_1, X_2, \dots$  are IID from a distribution having a variance  $\sigma^2$ , show that both  $V_n$  and  $S_n^2$  are consistent estimators of  $\sigma^2$ .

**3-7.** Suppose  $X_1, X_2, \dots$  are IID  $\text{Geo}(p)$ .

- (a) Find a method of moments estimator for  $p$ .
- (b) Find the asymptotic normal distribution of your estimator.

**3-8.** Suppose  $X_1, X_2, \dots$  are IID  $\text{Beta}(\alpha, 2)$ .

- (a) Find a method of moments estimator for  $\alpha$ .
- (b) Find the asymptotic normal distribution of your estimator.

**3-9.** Let  $X_1, X_2, \dots, X_n$  be an i. i. d. sample from a  $\text{Beta}(\theta, \theta)$  model, where  $\theta$  is an unknown parameter. Find a method of moments estimator of  $\theta$ .

## Review Problems from Last Year's Tests

**3-10.** For the following data

1.5   2.0   2.5   3.0   4.5

- (a) Find the mean of the empirical distribution.
- (b) Find the median of the empirical distribution.
- (c) Find  $P_n(X \leq 3)$  under the empirical distribution.
- (d) Find the 0.25 quantile of the empirical distribution.

**3-11.** Find the asymptotic distribution of the sample median of an IID sample from the  $\text{Exp}(\lambda)$  distribution.

**3-12.** Suppose  $X_1, X_2, \dots$  are IID  $\text{NegBin}(r, p)$ , where  $r$  is known and  $p$  is unknown.

- (a) Find a method of moments estimator for  $p$ .
- (b) Find the asymptotic normal distribution of your estimator.