

Name _____ Student ID _____

The exam is closed book and closed notes. You may use three $8\frac{1}{2} \times 11$ sheets of paper with formulas, etc. You may also use the handouts on “brand name distributions” and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 200. There are 7 pages and 8 problems.

1. [25 pts.] Suppose X_1, \dots, X_n are independent and identically distributed (IID) $\text{Exp}(\lambda)$ and the prior distribution for λ is $\text{Gam}(\alpha_0, \lambda_0)$, where α_0 and λ_0 are hyperparameters.

(a) Find the posterior distribution for λ .

(b) Find the mean of the posterior distribution for λ .

- (c) Find the variance of the posterior distribution for λ .
2. [25 pts.] Find the Jeffreys prior for the $\text{Geo}(p)$ distribution. It is proper or improper?

3. [25 pts.] The following Rweb output fits a linear model.

```
Rweb:> out <- lm(y ~ x + I(x^2))
Rweb:> summary(out)
```

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Call:
lm(formula = y ~ x + I(x^2))
```

Residuals:

Min	1Q	Median	3Q	Max
-1.6774	-0.4091	-0.0291	0.4301	2.5230

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.02422	0.08550	0.283	0.778
x	0.87653	0.07158	12.246	<2e-16 ***
I(x^2)	0.04954	0.05928	0.836	0.405

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6736 on 97 degrees of freedom

Multiple R-squared: 0.6086, Adjusted R-squared: 0.6005

F-statistic: 75.41 on 2 and 97 DF, p-value: < 2.2e-16

(a) Find a 95% confidence interval for the true unknown regression coefficient for the predictor $I(x^2)$. (Hint: The 0.95 quantile of the standard normal distribution is 1.645, and the 0.975 quantile of the standard normal distribution is 1.96.)

- (b) Perform a hypothesis test of whether this same regression coefficient is zero (the null hypothesis) versus nonzero (the alternative hypothesis), reporting and interpreting the P -value.
4. [25 pts.] Suppose X_1, \dots, X_n are IID $\text{Gam}(\alpha, \lambda)$, where both parameters are unknown. Show that $\sum_{i=1}^n X_i$ and $\prod_{i=1}^n X_i$ are components of a two-dimensional sufficient statistic.

5. [25 pts.] Suppose X_1, \dots, X_n are IID having PDF

$$f_{\theta}(x) = (\theta - 1)x^{-\theta}, \quad 1 < x < \infty$$

where $\theta > 1$ is an unknown parameter.

(a) Find the log likelihood for θ .

(b) Find the maximum likelihood estimate (MLE) for θ .

(c) Find the asymptotic distribution of your MLE.

6. [25 pts.] Suppose X is $\text{Geo}(p)$. We have only one observation. And suppose the prior distribution for p is proportional to p^{-1} , an improper prior.

(a) Find the posterior distribution for p .

(b) For what values of the data x does your answer to part (a) make sense?

7. [25 pts.] Suppose X_1, \dots, X_n are IID Cauchy($\theta, 1$). Find the asymptotic distribution of the sample median.

8. [25 pts.] Suppose X_1, \dots, X_n are IID Beta($\theta, \theta + 1$). where $\theta > 0$ is an unknown parameter. Find a method of moments estimator for θ .