Stat 5102 Second Midterm Exam **April 8, 2009**

Name ______________________________ Student ID ____________________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] Suppose X_1, …, X_n are IID $\text{Gam}(\alpha, \lambda)$, where α is known and λ is unknown.

   (a) Find the log likelihood for λ.

   (b) Find the maximum likelihood estimate (MLE) for λ.

(c) Show that your MLE is the unique global maximizer of the log likelihood.

(d) Find the expected Fisher information for λ.

2. [20 pts.] Suppose X_1, ..., X_n are IID having PDF

$$f_\theta(x) = \theta x^{\theta - 1}, \qquad 0 < x < 1$$

where $\theta > 0$ is an unknown parameter.

(a) Find the log likelihood for θ.

(b) Find the maximum likelihood estimate (MLE) for θ.

(c) Find the asymptotic distribution of your MLE.

(d) Find an asymptotic 95% confidence interval for θ. (Hint: The 0.95 quantile of the standard normal distribution is 1.645, and the 0.975 quantile of the standard normal distribution is 1.96.)

3. [20 pts.] Suppose X_1, …, X_n are IID $\text{Exp}(\lambda)$, and suppose the prior distribution for λ is $\text{Gam}(\alpha_0, \lambda_0)$, where α_0 and λ_0 are hyperparameters. Find the posterior distribution for λ.

4. [20 pts.] Suppose X is $\text{Poi}(\mu)$. We have only one observation. And suppose the prior distribution for μ is proportional to $\mu^{-1/2}$, an improper prior.

   (a) Find the posterior distribution for μ.

(b) For what values of the data x does your answer to part (a) make sense?

5. [20 pts.] Suppose X is $\text{Exp}(\lambda)$, where λ is an unknown parameter. We have only one observation.

   (a) Describe how to do an exact (not approximate) test of the hypotheses

$$H_0 \colon \lambda = \lambda_0$$
$$H_1 \colon \lambda < \lambda_0$$

   where λ_0 is a specified number (the value of λ hypothesized under H_0. Give a formula for the P-value of the test, an expression in terms of functions you can find on a calculator. (Hint: Consider the relationship between $E(X)$ and λ.)

(b) Calculate the P-value when $\lambda_0 = 1$ and $x = 3.7$.

(c) Interpret the P-value. What does it say about whether λ is larger or smaller than λ_0? What does it say about the statistical significance of this conclusion?