

Name _____ Student ID _____

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on “brand name distributions” and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 5 pages and 5 problems.

1. [20 pts.] The function

$$f_{\mu}(x) = \frac{2}{\pi(e^{x-\mu} + e^{-(x-\mu)})}, \quad -\infty < x < \infty$$

is a probability density function (PDF), where the parameter μ can be any real number. The mean and variance of this distribution are

$$\begin{aligned} E(X) &= \mu \\ \text{var}(X) &= \frac{\pi^2}{4} \end{aligned}$$

You do not have to prove any of the above. Given that information, find the asymptotic relative efficiency (ARE) of the sample mean and sample median of an independent and identically distributed (IID) sample from this distribution, both considered as estimators of μ .

2. [20 pts.] For the following data

1.5 2.0 2.5 3.0 4.5

(a) Find the mean of the empirical distribution.

(b) Find the median of the empirical distribution.

(c) Find $P_n(X \leq 3)$ under the empirical distribution.

(d) Find the 0.25 quantile of the empirical distribution.

3. [20 pts.] The function

$$f_{\mu}(x) = \frac{3\mu(\mu + 2)(2\mu + 1)x^{(4\mu-1)/(1-\mu)}(1-x)^2}{2(1-\mu)^3}, \quad 0 < x < 1,$$

is a PDF, where the parameter μ satisfies $0 < \mu < 1$. The mean and variance of this distribution are

$$\begin{aligned} E(X) &= \mu \\ \text{var}(X) &= \frac{\mu(1-\mu)^2}{4-\mu} \end{aligned}$$

You do not have to prove any of the above. Given that information, find an asymptotic 95% confidence interval for μ using an IID sample X_1, \dots, X_n from this distribution. Hint: the 0.975 quantile of the standard normal distribution is 1.9600.

4. [20 pts.] Find the asymptotic distribution of the sample median of an IID sample from the $\text{Exp}(\lambda)$ distribution.

5. [20 pts.] Suppose X_1, X_2, \dots are IID $\text{NegBin}(r, p)$, where r is known and p is unknown.

(a) Find a method of moments estimator for p .

(b) Find the asymptotic normal distribution of your estimator.