Stat 5102 (Geyer) Spring 2009
Homework Assignment 7
Due Wednesday, March 25, 2009

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

7-1. Suppose $X_1, \ldots, X_n$ are IID Cauchy(μ, σ) and $\sigma = 1$ is known. We wish to do maximum likelihood estimation, which cannot be done in closed form, so you must use R. One needs a good estimate of the location parameter to use as a starting point for the optimization. The location parameter is the center of symmetry and also the median. Thus the sample median is a good starting point. Data for the problem are at the URL

`http://www.stat.umn.edu/geyer/5102/data/prob7-1.txt`

(a) Find the MLE for these data.

(b) Find the observed Fisher information evaluated at the MLE.

(c) Find an asymptotic 95% confidence interval for the parameter μ.

7-2. Suppose $x_1, \ldots, x_n$ are known numbers (not random), and we observe random variables $Y_1, \ldots, Y_n$ that are independent but *not* identically distributed random variables having distributions

$$Y_i \sim \mathcal{N}(\alpha + \beta x_i, \sigma^2),$$

where α, β, and σ^2 are unknown parameters.

(a) Write down the log likelihood for the parameters α, β, and $\varphi = \sigma^2$.

(b) Find the maximum likelihood estimates of these parameters.

(c) Find the expected Fisher information matrix for these parameters.

Caution: In taking expectations remember only the Y_i are random. The x_i are known constants.

7-3. Suppose $X_1, \ldots, X_n$ are IID Laplace(μ, σ) and both parameters are considered unknown. We found the log likelihood in problem 6-4. Find the MLE of μ and σ.
Hint: For each fixed value of σ show that the sample median is the MLE of μ, the argument being as in problem 6-13. Call that MLE $\hat{\mu}_n$, now plug this estimate into the log likelihood obtaining a function

$$\sigma \mapsto l_n(\hat{\mu}_n, \sigma) \qquad (*)$$

which is a twice differentiable function of σ. Find a point where the first derivative of $(*)$ is zero, and show that it is a local maximizer. That point is the MLE for σ.

7-4. Suppose $X_1, \ldots, X_n$ are IID Cauchy(μ, σ) and both parameters are unknown. We wish to do maximum likelihood estimation, which cannot be done in closed form, so you must use R. One needs a good estimate of both parameters to use as a starting point for the optimization. As in problem 7-1 the sample median is a good starting point for μ. In 5101 problem 8-10(c) we found that the interquartile range (IQR) of the standard Cauchy distribution is 2. Thus the IQR of the Cauchy(μ, σ) distribution is 2σ, and half the IQR is a good starting point for σ. The R function `IQR` estimates the IQR. Data for the problem are at the URL

`http://www.stat.umn.edu/geyer/5102/data/prob7-1.txt`

(a) Find the MLE vector for these data.

(b) Find the observed Fisher information matrix evaluated at the MLE.

(c) Find asymptotic 95% confidence intervals for the parameters μ and σ. Do not adjust to obtain 95% simultaneous coverage.

7-5. Suppose $X_1, \ldots, X_n$ are IID Gam(α, λ) and $\lambda = 1$ is known. Suppose $n = 50$, the MLE is $\hat{\alpha}_n = 2.73$ and we wish to do test of the hypotheses

$$
\begin{aligned}
H_0 &: \alpha = 2.5 \\
H_1 &: \alpha \geq 2.5
\end{aligned}
$$

Find the asymptotic P-value for this test using the standardized MLE as the test statistic. You will have to use R to calculate Fisher information. Interpret the P-value.

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