Stat 5102 First Midterm Exam **March 1, 2000**

Name ______________________________ Student ID ____________________

The exam is open book, including handouts. It is closed notes. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 2 pages and 5 problems.

1. [15 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\mathcal{N}(3, 4)$ and $\overline{X}_n$ is the sample mean for a sample of size $n = 9$. Find $P(\overline{X}_n < 2)$.

2. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. from the distribution with density
$$f(x) = (\theta - 1)x^{-\theta}, \qquad x > 1,$$
where $\theta > 2$ is an unknown parameter. Find a method of moments estimator for θ.

3. [25 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\mathcal{U}(a, b)$, meaning they have p. d. f.
$$f_{a,b}(x) = \frac{1}{b - a}, \qquad a < x < b,$$
where a and b are unknown parameters.

   Hint:
$$\operatorname{var}(X_i) = \frac{(b - a)^2}{12}$$

   (a) Find the asymptotic distribution of $\overline{X}_n$.

   (b) Find the asymptotic distribution of $\widetilde{X}_n$.

   (c) Find the asymptotic relative efficiency of $\overline{X}_n$ with respect to $\widetilde{X}_n$ when both are considered estimators of the parameter $\theta = (a + b)/2$. Also state which is the better estimator.

4. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\mathcal{N}(\mu, \sigma^2)$ distribution, where μ and σ^2 are unknown parameters, and suppose S_n^2 is the usual sample variance given by

$$S_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X}_n)^2.$$

Find an exact 90% confidence interval for σ^2 corresponding to data $n = 10$ and $S^2 = 2.2$.

5. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\mathrm{Exp}(\lambda)$ random variables and, as usual, $\overline{X}_n$ denotes the sample mean. What is the asymptotic distribution of

$$Y_n = e^{-t/\overline{X}_n},$$

where t is a known positive number?