Statistics 5101, Fall 2000, Geyer

Homework Solutions #2

Note: Original done by Laura Pontiggia, Fall 1999. Additions by Yumin Huang, Fall 2000.

Problem 2-12

Since $(A \cup B^c)^c = A^c B$, and since $\text{pr}(B) = \text{pr}(A^c B) + \text{pr}(AB)$ [using prop. (7)] then $\text{pr}(A^c B) = \text{pr}(B) - \text{pr}(AB) = .30 - .21 = .09$. Using prop. (1): $\text{pr}(A \cup B^c) = 1 - \text{pr}[(A \cup B^c)^c] = 1 - .09 = .91$.

Problem 2-20

The inequality on the right (the only one we were asked to do) $\text{pr}(E \cup F) \leq \text{pr}(E) + \text{pr}(F)$ follows from Property (3) and Axiom 1.

Problem 2-22

(b) By Theorem 2, $\text{pr}(E_1 \cup E_2) = \text{pr}(E_1) + \text{pr}(E_2) - \text{pr}(E_1 E_2)$. Since $\text{pr}(E_1 E_2)$ is nonnegative. The inequality holds for $n = 2$.

Now, assume the inequality holds when $n = k - 1$. Let $F = E_1 \cup E_2 \cup \cdots \cup E_{k-1}$. Applying Theorem 2(3) again, it follows that $\text{pr}(F \cup E_k) \leq \text{pr}(F) + \text{pr}(E_k)$. Therefore the inequality holds by mathematical induction.

Problem 2-26

Since each characteristic two possible values, the number of outcomes in this experiment is $4^2 = 16$.

The probability distribution that reflects the proportions 9:3:3:1 is $(\frac{9}{16}, \frac{3}{16}, \frac{3}{16}, \frac{1}{16})$. Hence the probability that a plant has yellow peas is given by

$$\text{pr(a plant has yellow peas)} = \text{pr(WY)} + pr(\text{RY}) = \frac{3}{16} + \frac{9}{16} = \frac{3}{4}.$$

Problem 2-27

(a)

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{i+j} = \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^{i} \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j} = 1 \times 1 = 1.$$

(b) Call the event in question A, so

$$A = \{ (1,1), (1,2), (2,1), (1,3), (3,1), (2,2) \}.$$

Then

$$\text{pr}(A) = \frac{1}{2^{1+1}} + \frac{2}{2^{1+2}} + \frac{2}{2^{3+1}} + \frac{1}{2^{2+2}} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} = \frac{11}{16}.$$

Problem 3-3

(a)

X \ Y	1	2	3	4	$f(x)$
1	0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
2	$\frac{1}{12}$	0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
3	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{12}$	$\frac{1}{4}$
4	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{4}$
$f(y)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1

The distribution is uniform (equally likely outcomes) except for the outcomes on the diagonal (with $X = Y$), which are impossible because of the sampling "without replacement."

(b)

X	1	2	3	4
$f_X(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Y	1	2	3	4
$f_Y(y)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(c)

Z	3	4	5	6	7
$f(z)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\text{pr}(Z = X + Y = 3) = \text{pr}(X = 1, Y = 2) + \text{pr}(X = 2, Y = 1) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}.$$

$$\text{pr}(Z = X + Y = 4) = \text{pr}(X = 1, Y = 3) + \text{pr}(X = 3, Y = 1) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}.$$

$$\begin{aligned}\operatorname{pr}(Z = X + Y = 5) &= \operatorname{pr}(X = 1, Y = 4) + \operatorname{pr}(X = 4, Y = 1) \\ &\qquad + \operatorname{pr}(X = 2, Y = 3) + \operatorname{pr}(X = 3, Y = 2) \\ &= \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{3}.\end{aligned}$$

and so forth for $\operatorname{pr}(Z = 6)$ and $\operatorname{pr}(Z = 7)$.

Problem 3-4

There are six points in the sample space

guess	number correct
A B C	3
B C A	0
C A B	0
C B A	1
B A C	1
A C B	1

Thus

X	0	1	2	3
$f(x)$	$\frac{1}{3}$	$\frac{1}{6}$	0	$\frac{1}{2}$

(or the point 2 can be deleted from the sample space if you prefer).

Problem 3-6

There are $\binom{12}{3}$ points in the sample space (ways to choose 3 eggs from 12 eggs).

Similarly, there are are $\binom{2}{k}$ to choose k rotten eggs from the 2 rotten eggs in the carton and $\binom{10}{k}$ to choose k non-rotten eggs from the 10 non-rotten eggs in the carton. If there are k rotten eggs drawn, there are $3 - k$ non-rotten eggs drawn. Thus

$$f(y) = \frac{\binom{2}{y}\binom{10}{3-y}}{\binom{12}{3}}, \qquad y = 0, 1, 2.$$