Statistics 5101, Fall 2000, Geyer
Homework Solutions #1

Note: Original done by Laura Pontiggia, Fall 1999.

Problem 1-2

Since each function can be defined to have any one of k values in any one of m points in the domain, the number of outcomes in this experiment is

$$\underbrace{k \cdot k \cdot \ldots \cdot k}_{m \text{ terms}} = k^m$$

Problem 1-4

(a) The multiplication rule will give a count of ordered sequences. Since each die can turn up in one of six ways, the number of outcomes in the composite experiment is

$$6 \cdot 6 \cdot 6 = 216.$$

(b) Since each coin can turn up in one of two ways (heads or tails), the number of outcomes in this composite experiment is 2^n.

(c) Because the positions are distinguished, we need to count the number of ordered arrangements of four of the ten persons. We choose one of the 10 persons to serve as president, then we choose one of the remaining 9 persons to serve as vice-president, then we choose one of the remaining 8 persons to serve as secretary, and finally we choose one of the remaining 7 persons to serve as treasurer.

The count is

$$(n)_k = \frac{n!}{(n-k)!} = \frac{10!}{(10-4)!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

Problem 1-5

(a) Since each symbol can be defined in one of two ways ($+$ or $-$), the number of outcomes in this experiment is $2^{10} = 1024$.

(c) We divide the ten symbols into two groups: one groups of five $+$'s and another of five $-$'s.

The number of sequences in which we have exactly five $+$'s and five $-$'s is the same as the number of ways in which we can choose five positions out of the ten positions to place $+$'s

$$\binom{10}{5} = \frac{10!}{5!5!} = 252.$$

(e) The sequences of first five symbols must be contain two $+$'s and three $-$'s, and the other sequence must be contain three $+$'s and two $-$'s. Then the number of outcomes is

$$\binom{5}{2}\binom{5}{3} = 100.$$

Problem 1-8

(a) When a hand of five card is dealt from a deck of playing cards, the number of possible distinct poker hands is

$$\binom{52}{5} = 2598960.$$

(b) The number of possible ways to choose five cards in the hearts suit is:

$$\binom{13}{5} = 1287.$$

(c) If we suppose that the Ace can be just high, we have nine possible unbroken sequences of 5 cards (Ace low to nine low), and each denomination has 4 suits. Then the number of possible outcomes of this experiment is

$$9 \cdot 4^5 = 9216.$$

This is the answer in the book, but it is not the right answer. Lindgren doesn't know how to play poker.

The correct answer is that Ace can be high or low (A, 2, 3, 4, 5 is the lowest straight and 10, J, Q, K, A is the highest). So we have ten possible unbroken sequences of 5 cards, and the number of possible outcomes is

$$10 \cdot 4^5 = 10240.$$

Problem 1-25

(a) Same as the number of ways to choose 2 places to put the letter H out of 4 places, which is $\binom{4}{2} = 6$.

(b)

$$\binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 6 + 4 + 1 = 11$$

Problem 1-26

(a)

$$(5)_2 = 20$$

(order matters, we can distinguish the first and second chip).

(b) There are 4 possibilities for the other chip.

(c) Same as (b). This is easy if looked at the right way, hard if looked at any other way. Time is irrelevant. The chips could be selected in a different order, "second" chip first. That makes (b) and (c) the same problem.

(d) The sum of (b) and (c), because the events are disjoint (the first and second chips cannot both be 4).

(e)

$$(4)_2 = 12$$

same as part (a) except with only 4 chips (excluding the 4).

Problem 2-2

(a) Consider the following table of the sums that can occur (all outcomes equally likely)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We have 9 outcomes in which the sum is divisible by 4. So

$$\text{pr}(\text{sum is divisible by 4}) = \frac{9}{36} = \frac{1}{4}.$$

(b) Consider the following table of the combinations of sides that can occur.

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

We have 9 outcomes in which both numbers are even. So

$$\text{pr(both numbers are even)} = \frac{9}{36} = \frac{1}{4}.$$

(c) We use the last table. We have 6 outcomes in which the numbers match (1,1), (2,2) ..., (6,6). So

$$\text{pr(the numbers match)} = \frac{6}{36} = \frac{1}{6}.$$

(d) Use same table as in (b). The cases in which the numbers differ by at least 4 are: (1,5), (1,6), (2,6), (6,1), (6,2), (5,1). We have 6 outcomes in which the numbers differ by at least 4. So

$$\text{pr(the numbers differ by at least 4)} = \frac{6}{36} = \frac{1}{6}.$$

(e) We use table as in (a). We have 3 outcomes in which the sum is more than 10 (i. e., 11, 11, 12). Then we have $(36 - 3) = 33$ outcomes in which the sum is at most 10. So

$$\text{pr(the sum is at most 10)} = \frac{33}{36} = \frac{11}{12}.$$

Problem 2-4

(a) Every outcome is equally likely to occur, so C has one chance on 4 of getting his or her own hat. So

$$\text{pr(C gets his or her own hat)} = \frac{1}{4}.$$

(c) Note that if B, C, and D get their own hats, then so does A. So there is only one way for this to happen. The cardinality of the sample space is the number of permutations of four objects, which is $4! = 24$. So

$$\text{pr(B, C, and D get their own hats)} = \frac{1}{4!} = \frac{1}{24}.$$

Problem 2-6

We have 12 contestants, so 12 outcomes. Each outcome is equally likely to occur, so if we pick 4 winners the probability that I am one winner is

$$\frac{4}{12} = \frac{1}{3}.$$

Problem 2-9

(a) The pair may be from any one of 13 denominations, and is two of a possible 4 cards. Once the pair has been chosen, the remaining 3 cards can have any distinct 3 of the remaining 12 denominations, and each is one out of 4 cards in that denomination.

The number of total outcomes is

$$\binom{52}{5} = 2598960.$$

The number of outcomes in which we have exactly one pair is

$$13\binom{4}{2}\binom{12}{3}4^3 = 1098240.$$

So

$$\text{pr(the hand contains exactly one pair)} = \frac{1098240}{2598960} = 0.423$$

(c) Already done in class and as Example 2.1b in Lindgren.