Stat 5101 Second Midterm Exam   **November 12, 2014**

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The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may use the handout on "brand name distributions". You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from. Leave no undone integrals in your answers, but other than that requirement there is no unique "correct" simplification. Any correct answer gets full credit.

Points for the questions total to 100. There are 5 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having probability density function (PDF) given by

$$f_\theta(x) = \frac{1}{x \log(\theta)}, \qquad 1 < x < \theta,$$

where $\theta \geq 1$ is a parameter.

(a) Find the mean of the distribution of X.

(b) Find the variance of the distribution of X.

2. [20 pts.] Suppose X is a random variable having a $\text{Gam}(\alpha, \lambda)$ distribution. Find the PDF of the random variable $Y = 1/(1 + X)$. The definition of a function describes the domain as well as the rule.

3. [20 pts.] Suppose X is a random variable having PDF given by

$$f_\theta(x) = \frac{\theta e^{\theta x}}{e^\theta - e^{-\theta}}, \qquad -1 < x < 1,$$

where $\theta > 0$ is a parameter. Find its distribution function (DF). Be sure to define the DF on the whole real line.

4. [20 pts.] Suppose the random vector (X, Y) has the PDF

$$f_\theta(x, y) = \frac{6(\theta x + y)^2}{2 + 3\theta + 2\theta^2}, \qquad 0 < x < 1, \ 0 < y < 1,$$

where $\theta > 0$ is a parameter.

(a) Find the conditional PDF of Y given X.

(b) Find the conditional expectation of Y given X.

5. [20 pts.] Suppose the conditional distribution of Y given X is $\mathrm{Geo}(X)$, and suppose the marginal distribution of X has PDF

$$f_\theta(x) = \theta x^{\theta-1}, \qquad 0 < x < 1,$$

where $\theta > 0$. What is the conditional distribution of X given Y? Since this is a brand name distribution, no integrals need be done, it is enough to name the distribution and give its parameters as a function of y and θ.