Stat 5101 First Midterm Exam **October 10, 2012**

Name _____ Student ID _____

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handout on "brand name distributions". Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 7 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having probability mass function (PMF) given by

x	1	2	3	4	5
$f(x)$	1/4	1/4	1/6	1/6	1/6

   (a) Calculate $E(X)$.

   (b) Calculate $\text{var}(X)$.

(c) Calculate $\Pr(|X-3|>1)$.

2. [20 pts.] Suppose X is a random variable having PMF given by

x	1	2	3	4	5	6
$f(x)$	1/12	2/12	3/12	3/12	2/12	1/12

Find the PMF of the random variable $Y=(X-2)(X-5)$.

3. [20 pts.] Suppose X_1, X_2, and X_3 are exchangeable random variables all having mean μ and variance σ^2 and all pairs having covariance γ. Find the mean vector and variance matrix of the random vector

$$\mathbf{Y} = \begin{pmatrix} X_1 \\ X_1 - X_2 \\ X_1 + X_2 + X_3 \end{pmatrix}$$

4. [20 pts.] Suppose the random vector (X, Y, Z) has PMF given by each of the following definitions. In each part say whether X, Y, and Z are independent random variables, and explain why or why not, as the case may be. In all parts of this question $S = \{1, 2, 3, 4, 5\}$.

   (a)
   $$f(x, y, z) = \frac{2^{15-x-y-z}}{29791}, \qquad (x, y, z) \in S^3$$

   (b)
   $$f(x, y, z) = \frac{2^{15-x-y-z}}{11811}, \qquad (x, y, z) \in S^3 \text{ and } x \le y \le z$$

(c)

$$f(x,y,z) = \frac{15-x-y-z}{750}, \qquad (x,y,z) \in S^3$$

(d)

$$f(x,y,z) = \frac{15-x-y-z}{210}, \qquad (x,y,z) \in S^3 \text{ and } x \leq y \leq z$$

(e)

$$f(x,y,z) = \frac{\log(xyz)}{c}, \qquad (x,y,z) \in S^3,$$

where

$$c = \sum_{(x,y,z)\in S^3} \log(xyz)$$

(f)

$$f(x,y,z) = \frac{\log(xyz)}{c}, \qquad (x,y,z) \in S^3 \,\text{and}\, x \le y \le z,$$

where

$$c = \sum_{\substack{(x,y,z)\in S^3 \\ x\le y\le z}} \log(xyz)$$

5. [20 pts.]

   (a) Suppose 5 cards are dealt from a well shuffled deck. What is the probability that at least 4 cards are red? (There are 52 cards in a deck, 26 are red, and 26 are black. Dealing cards from a deck means sampling *without replacement*.)

   (b) Suppose 5 cards are dealt from a deck, but unlike the usual procedure, each card dealt is put back in the deck and the deck reshuffled between deals (that is, this is now sampling *with replacement*). What is the probability that at least 4 cards are red?